ESD-TR-91-211 MTR-11120

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Mobile Microwave Landing System (MMLS): Operational Requirements for Setup Accuracy

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August 1991

Prepared for
Microwave Landing System Program Manager
Electronic Systems Division
Air Force Systems Command
United States Air Force
Hanscom Air Force Base, Massachusetts

Project No. 5420
Prepared by
The MITRE Corporation
Bedford, Massachusetts
Contract No. F19628-89-C-0001

REVIEW AND APPROVAL

This technical report has been reviewed and is approved for publication.

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
August 1991

3. REPORT TYPE AND DATES COVERED
Final

4. TITLE AND SUBTITLE
Mobile Microwave Landing System (MMLS): Operational Requirements for Setup Accuracy

5. FUNDING NUMBERS
F19628-89-C-0001
5420

6. AUTHOR(S)
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Powell, Frederic D.

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
The MITRE Corporation
Burlington Road
Bedford, MA 01730

8. PERFORMING ORGANIZATION REPORT NUMBER
MTR-11120

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Microwave Landing System Program Manager (ESD/TGM)
Electronic Systems Division, AFSC
Hanscom AFB, MA 01731-5000

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
ESD-TR-91-211

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
This report examines, and determines, the accuracies required for the survey and alignment procedures in operational use of the Mobile Microwave Landing System such that the specified system accuracies will be satisfied in collocated and split-site Category I and II system deployments. The required accuracies are within the capabilities of modern survey equipment; in fact, less sophisticated methods such as pacing, or using the known length of the monitor cables, will suffice.

14. SUBJECT TERMS
Mobile Microwave Landing System
Setup Accuracy
Survey

15. NUMBER OF PAGES
65

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	SAR

ACKNOWLEDGEMENT

This document has been prepared by The MITRE Corporation under Project No. 5420, Contract No. F19628-89-C-0001. The contract is sponsored by the Electronic Systems Division, Air Force Systems Command, United States Air Force, Hanscom Air Force Base, Massachusetts, 01731-5000.

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EXECUTIVE SUMMARY

The operational uses of the Mobile Microwave Landing System (MMLS) are as a temporary replacement for a permanent installation which is out of service, or in tactical applications in the field. In either application, and especially in tactical use where a pre-surveyed site cannot be assumed, the ability to set up the system quickly is critical. The speed of the procedure, the time within which the setup can be completed, depends on a variety of elements; this report concentrates on one aspect – the accuracy required of the survey and angular alignment. This report examines the accuracy required for the survey and alignment, collectively the setup, in four deployment scenarios. These four deployment scenarios are:

1. The ground units are collocated at the conventional location for the elevation antenna, about 800 to 1000' towards the stop-end of the runway, and 150 off the centerline. Category I (Cat I) operation is required;

2. The ground units are collocated about 200' upwind from the threshold, and 150' offset, a short-field arrangement, and Cat I operation is required;

3. The azimuth antenna is located on the runway centerline extension at the stop-end of a 12,000' runway, the elevation antenna is sited as in Case 1 except 450' offset from the centerline, and Cat I operation is required.

4. The ground units are sited as in Case 3, and Cat II operation is required.

For each case, the random errors inherent in the equipment are combined, RSS, and this random value is subtracted from the allowable system errors for the two categories of operation. The difference, or margin, is assigned the budgets of the various survey and alignment elements. The problem is mathematically underdetermined: there are many variables to be considered but only two equations (lateral and vertical margins). However, many of the variables in the several equations for each case have very small coefficients, and therefore can be neglected; this reduces the number of degrees of freedom, and enables trade-offs, so that the accuracy requirements for the linear and angular setup elements can be met simultaneously.

Table ES-1 presents the most critical elements of the budgets for the four cases. The accuracies of the linear setup, the survey, are within the capability of a crew using the monitor cables as measuring tapes. In operational situations, the required linear survey accuracies can

Table ES-1

Required Accuracies of Setup

	Parallel to centerline	Across centerline	Vertical	Angular Alignment
Case 1	49'	3.3'	3.3'	0.20 degrees
Case 2	49'	2.9'	3.3'	0.20 "
Case 3	49'	1.5'	3.3'	0.20 "
Case 4	10'	0.5'	2.3'	0.15 "

be accomplished by pacing to establish distances. Similarly, the angular alignment budget is three (or four) times the accuracy of the built-in inclinometers. It therefore is concluded that both the linear and angular aspects of the setup can be accomplished without special difficulties, and specialized survey equipment, such as a surveyor's transit or a theodolite, are not required for accurate and rapid setup.

In table ES-4, Case 4 shows the most severe requirement for the linear survey; this is a budget of 1/2 foot for the lateral location of the azimuth antenna. This accuracy depends on the operational procedure used to emplace the azimuth antenna, and also to emplace its sighting pole and thus its monitor. Two procedures may be envisioned:

a. A location is selected for emplacing the azimuth antenna; in Case 4, this is on the extension of the runway centerline. The accuracy requirement is stated in table ES-1 as 1/2 foot. The azimuth sighting pole is then similarly, but independently, emplaced on the runway centerline extension; the same accuracy of location for the sighting pole is required namely, 1/2 foot. The accuracy of the two emplacements is required to ensure that the angular error of alignment of the azimuth antenna in yaw does not exceed the available margin of the Case. This procedure uses a measuring tape, or the monitor cables as an alternate thereof, as the measuring tools.

b. An alternate procedure uses a theodolite or transit to find the correct location for the azimuth sighting pole. In this case, required accuracy for the siting of the azimuth antenna becomes much larger, 15 feet in the critical Case 4.

The first of these two options was selected as the basis for this study, to minimize the list of special equipments required for a tactical deployment.

However, if the survey is performed somewhat more accurately than shown in table ES-1, the budgets for the angular alignments become much larger, and the system can operate within the specified tolerances, despite a variety of situations. In particular, the effects of frost heaves in unprepared sites can be disregarded until the alignment errors become so large that the equipment shuts off automatically.

SECTION I

INTRODUCTION

The uses of the mobile microwave landing system (MMLS) are as a temporary replacement of a fixed base system that is out of commission, or to support tactical military missions. In the latter functions, it may be necessary to install the system at a site which has not previously been surveyed. The accuracy of the survey and of the alignment of the antennas – collectively, the setup – then contribute significantly to the errors of the position of the aircraft with respect to the desired flight path at the various decision heights. This report examines the effects of setup accuracy on the system performance in the operational context, and finds, to the extent possible, the errors of setup which can be allowed without degrading the system accuracy specified for several deployment situations.

As defined by the International Civil Aeronautics Organization (ICAO) in reference 1, the microwave landing system (MLS) in principle is comprised of three separate ground units plus cooperative avionics. The three ground units are an azimuth antenna, an elevation antenna, and a distance measuring equipment (DME) transponder. The avionics decode signals from the ground units to enable locating the aircraft relative to the runway, and to establish guidance information for the pilot. The MMLS differs from the ICAO definition in that the DME and the azimuth antenna are physically joined, and thus are always collocated. Two siting arrangements are anticipated with the MMLS. The conventional split-site arrangement places the azimuth antenna and the DME transponder on or near the runway centerline at the stop-end, while the elevation antenna is located near the intended touchdown point about one thousand feet upwind from the threshold. A collocated configuration is also possible in which all the ground units are at the elevation antenna site.

Four situations, two collocated and two split, are considered for evaluation in this report. One of the collocated cases assumes the aircraft to be at the Category I decision height (Cat I DH) of 200', and the ground units at the usual location of the elevation antenna, about 800 to 1000' upwind from the threshold, so that the aircraft passes over the threshold at the conventional height of 50'. The other collocated case assumes that the runway is very short, and therefore places the ground units very close to the threshold, only 200' upwind. In the two split-site cases, the MMLS azimuth antenna/DME combination is sited on the runway centerline at the stop-end of a 12000' runway, but the aircraft is assumed to be at the Cat I DH of 200' and at the Cat II DH of 100', respectively.

In each of the situations mentioned above, the total random errors are determined and are compared to the required accuracy for that situation. The remainder, the margin, is then assigned to setup errors, which are arbitrarily, and perhaps unrealistically, assumed to be exactly and positively correlated. This approach, yielding a worst-case measure of the error of aircraft position, is discussed further in section 3. A matrix of sensitivity coefficients relates the error margin components to the specific components of setup accuracy. A budget for the allowable errors of the individual components of the survey and alignment procedure is established for each case.

However, the specifications of required system performance do not enable determination of every component of the setup, for there are more unknown variables than there are equations: the system is mathematically under-determined. Engineering judgement is therefore used where required.

Notation is presented in section 2. The "Collocated Cat I" case is studied in section 3, and the "Collocated Short-Field Cat I" case in section 4; the former is used as the format for all of the analyses, which therefore are somewhat abbreviated. The "Split-Site Cat I" case is studied in section 5, and the "Split-Site Cat II" case in section 6. The results are gathered and discussed in section 7 ; conclusions are presented in section 8. The mathematical development of the coefficients that show the sensitivity of the position errors to survey and alignment errors is presented in appendix A. The combined effects of the DME error and the guidance principle when the azimuth antenna is offset from the runway centerline are considered in appendix B.

SECTION 2

GEOMETRY, NOTATION, AND MATHEMATICS

This section presents the geometry, notation, and the mathematical foundations for the MMLS survey accuracy problem.

The coordinate system for the problem is defined in figure 2-1. The origin of the coordinates is the runway threshold. The MMLS x-axis is selected to be the runway centerline and its extension, with negative values toward the stop-end of the runway, so that any conventional location for the ground units has a negative x-value. An azimuth antenna located near the stop-end of a 5000' runway thus has an x-value of about $x_A = -5000'$, where the subscript A implies azimuth antenna. The positive direction of y lies to the left of an observer who is standing at the origin with the stop-end of the runway behind. The positive direction of z is up. The elevation angle is defined as positive counterclockwise, looking in the positive direction along the y-axis, so that positive elevation angles correspond to positive altitude. Azimuth is defined as positive clockwise from the x-axis, about the z-axis, looking down towards the ground. The angular coordinate system is not right-handed, but conforms to that of references 1 and 2, and to air navigation conventions. The azimuth antenna boresight is assumed, for simplicity but without loss of generality, to be parallel to the runway centerline; a well-known rotation and de-rotation enable treating other orientations.

Figure 2-1 shows a general split-site situation, appropriate to the MLS, where the three ground-units may be separate. In the MMLS the DME is mounted on the azimuth antenna, and thus is necessarily collocated therewith.

The runway is assumed to be straight, flat, and horizontal.

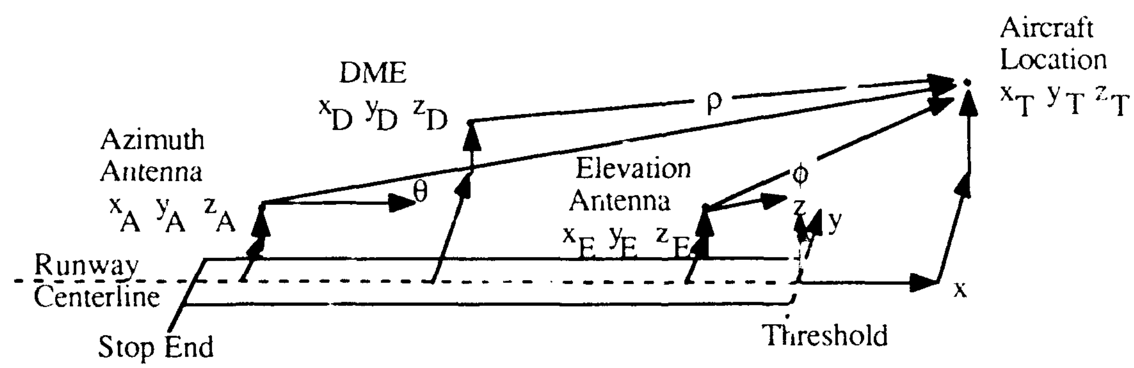

Figure 2-1. Geometry

The ground units' site data, defined below, are transmitted to the aircraft.

x_A, y_A, z_A	Components of position of azimuth antenna
x_D, y_D, z_D	Components of position of the DME
x_E, y_E, z_E	Components of position of the elevation antenna

The errors of the survey are denoted as

x_S, y_S, z_S	Survey errors

The aircraft position is

x_T, y_T, z_T	Components of position of the aircraft

The avionics observations are: slant range ρ, elevation angle ϕ, azimuth angle θ.

Slant Range

$$\rho = \sqrt{(x_T - x_D)^2 + (y_T - y_D)^2 + (z_T - z_D)^2} \qquad (2\text{-}1)$$

Azimuth angle

$$\theta = \arctan \left\{-(y_T-y_A)/\sqrt{(x_T-x_A)^2 + (z_T-z_A)^2}\right\} \qquad (2\text{-}2)$$

Elevation angle

$$\phi = \arctan \left\{(z_T-z_E)/\sqrt{(x_T-x_E)^2 + (y_T-y_E)^2}\right\} \qquad (2\text{-}3)$$

The analysis uses the slant range from the DME and also the slant range from the elevation antenna. The former is identical to (2-1); the notation is

ρ_{DA}	Slant range from DME and azimuth antenna to aircraft
ρ_E	Slant range from elevation antenna to aircraft

The angular orientations of the azimuth and elevation antennas are defined relative to the runway centerline and the horizontal plane by

α_Y, α_P, α_R	Azimuth antenna yaw, pitch, roll
β_Y, β_P, β_R	Elevation antenna yaw, pitch, roll

A few special symbols and notation are used in the appendices; these are defined where they are introduced.

SECTION 3

CASE 1: COLLOCATED EQUIPMENTS, CAT I PERFORMANCE

This section examines the error sources and errors for Cat I performance with the ground units collocated at the location for the elevation antenna. The lateral (cross-runway) and altitude aspects are examined to determine the allowable error budget for the setup. The required survey accuracy will then be determined. The error sources are the DME system, the azimuth and elevation antennas and their associated electronics, the avionics, the quantization of the transmitted ground unit site data, and the setup. Errors due to such sources as rain, multipath, and other elements of the environment are subsumed within the specifications for Path Following Error (PFE) and are therefore not considered separately hereunder. The avionics errors are expressed in terms of PFE; the values used are from references 1, 2, and 3. The errors of concern are usually random and uncorrelated, and there is no physical reason to suggest otherwise; the analysis therefore treats them accordingly. On the other hand, the setup errors could flow from common sources in the survey equipment or from repeated errors of procedure in the setup; the setup errors are therefore assumed to be correlated. This is not a statistically reasonable assumption; it is as conservative as possible. Under this assumption, the survey errors' magnitudes are added; this produces a bias of which only the positive or negative direction is random, and hence a worst-case result. Evaluation of the setup errors which may be allowed is considered at the end of the section.

The conditions which underly this case are:

a. Glide slope= 3°, DH = 200';
b. All ground units collocated at the normal elevation antenna site adjacent to the touchdown-point as shown in figure 3-1;
c. Phase center height of all units above ground = 5'; lateral offset = 150';
d. Aircraft is assumed to be on the runway centerline extension.

These conditions enable specifying the location of the collocated units, and the other elements of the geometry. The elevation antenna generates a cone, radially symmetrical about a vertical axis. The MLS approach reference datum (ARD) is 50' above the threshold; therefore, the conical elevation antenna pattern (2-3) requires that the ground units be at

$$z = \text{ARD} = 50 = z_E + \sqrt{(x_T-x_E)^2+(y_T-y_E)^2}\ \tan 3° \qquad (3\text{-}1)$$

At the ARD, $x_T = y_T = 0$, $y_E = y_D = 150'$, and $z_E = 5'$. Then (3-1) yields $x_E = -845'$ as the location of the ground units. Evaluating (3-1) at aircraft altitude of $z_T = 200'$ yields $x_T = 2872'$ and therefore $(x_T - x_E) = 3717'$. The geometry for all four cases is defined and evaluated fully in table A-1 in appendix A. As the elevation antenna pattern is conical, a flight path parallel to the runway centerline does not generate a path which is a straight line in altitude; the vertical path is a section of a hyperbola asymptotic to the 3° straight line.

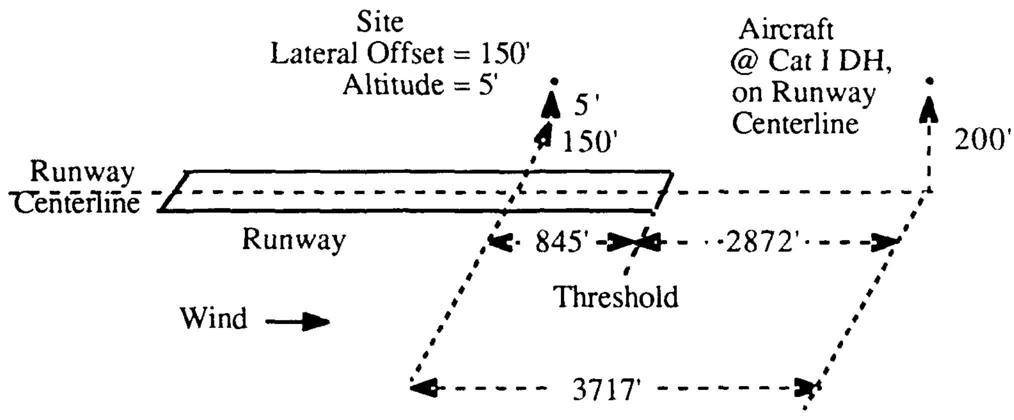

Figure 3-1. Geometry of Case 1

Errors of the lateral degree of freedom are considered in subsection 3.1; those of the vertical degree of freedom are in subsection 3.2. The allowable survey and alignment errors are determined in subsection 3.3.

3.1 LATERAL

The random contributors to the lateral errors are the DME, the azimuth antenna with its electronics, the avionics, and the error of quantization of the data. As discussed above, the survey errors are treated as non-random biasses, and other sources of error are neglected.

3.1.1 Error Due to the DME System

The DME system herein considered consists of a DME/P ground unit transponder and a DME/N airborne transponder; the geometry is also important in determining the lateral error due to the DME. The analysis of the effect of DME error on lateral position error, including the effects of the guidance law, is presented in appendix B. From (B-6), the lateral error of the aircraft position is

$$\varepsilon_D = |y_D[\Delta\rho/(\rho + \Delta\rho)]| \tag{3-2}$$

where $y_D = 150'$

ρ = slant range= 3725', as shown in table A-1

$\Delta\rho$ = DME range error = 644', as shown below.

The DME range error, $\Delta\rho$, is determined as follows. The ground-based DME/P transponder PFE is 50', as per table C-6 (IA Mode 3) of reference 1; the TACAN interrogator error is 608' as per 3.2.1.3.7.2 of reference 4; the propagation error is 206', as per Table C-6 of reference 1; these combine, RSS, to yield a value of $\Delta\rho$= 644', so that evaluating (3-2) at

the Cat 1 DH yields $\varepsilon_D = 150(644)/(3725 + 644) = 22.1'$ if $\Delta\rho > 0$, and $\varepsilon_D = 31.4$ if $\Delta\rho < 0$. Taking the latter, as it is more critical, the lateral error is

$$\varepsilon_D = 31.4' \tag{3-3}$$

at the Cat 1 location. This error, ε_D, increases as distance from the elevation antenna decreases, as shown in figure 3-2, based on (3-2); its value is 31.4' at the Cat I location for this case. It will be treated as Mean Course Error (MCE), and will then be combined, RSS, with the other errors.

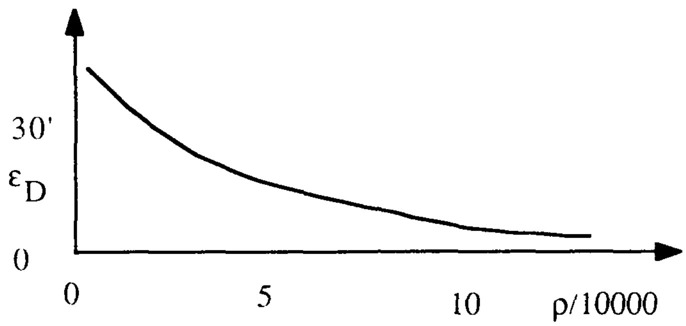

Figure 3-2. Error Due to DME as a Function of Range

3.1.2. Azimuth Antenna and Electronics

Mean Course Error (MCE). From 3.2.1.13 of reference 5, the azimuth antenna MCE is 0.060°. Other error sources discussed in 3.2.1.6,7 and 8 (beam steering accuracy and settling time, levelling, and electronic boresighting) are subsumed within this value. As the slant range from the ground units is 3725', from table A-1, the MCE at the Cat 1 DH is

$$\text{MCE} = 3725 \sin 0.06° = 3.9' \tag{3-4}$$

Path Following Noise (PFN). From 3.2.1.13 of reference 5, the PFN is 0.02°. The slant range from the collocated equipments to the aircraft is 3725'. The error due to PFN is thus

$$\text{PFN} = 3725 \sin 0.02° = 1.3' \tag{3-5}$$

3.1.3. Lateral Quantization Error

The least significant bit with which the x- and y-direction components of location of the ground units are encoded is 1 meter, as per reference 1. The maximum error is thus 1/2 meter. It is assumed that the actual error is uniformly distributed across this interval. The 2-σ error for a uniform distribution of (1/2) meter is $2(1/2)/\sqrt{3} = 0.577$ meters = 1.9'. This value is conservative, exceeding the maximum quantizing error.

3.1.4 Level Sensor Errors

From 3.7.9 of reference 5, the azimuth antenna roll and pitch alignment sensors have errors of 0.05°. From (A-25-1), the former contributes $3.4\Delta\alpha_R$ to the lateral error, while the latter has no first-order effect. The error contribution is thus 3.4(0.05) = 0.2'. The azimuth antenna yaw is controlled by a closed loop involving a monitor, and its error is contained within the MCE stated above.

3.1.5 Avionics Error

The slant range is 3725', and the avionics PFE is 0.017°; separation of MCE and PFN is not required. This error source therefore contributes 3725sin0.017 = 1.1'.

3.1.6. Total Lateral Error and Error Margin

The MCE, PFN, and level sensor errors evaluated above are combined RSS as they are statistically independent, as discussed above. The lateral error contributed by the DME system is statistically independent, although the equipments are physically collocated. Its contribution is therefore also added RSS. The total lateral error of the system in this Case 1 is

$$\text{Total Lateral Error, RSS} = \sqrt{31.4^2 + 3.9^2 + 1.3^2 + 0.2^2 + 1.1^2 + 1.9^2} = 31.7' \quad (3\text{-}6)$$

The allowable lateral error is 65', and the survey and setup margin is

$$\text{Margin} = 65 - 31.7 = 33.3' \quad (3\text{-}7)$$

3.2 VERTICAL

In the present problem, the elevation antenna is collocated with the azimuth/DME combination. Many of the geometric elements which were deduced for the lateral problem will be applicable. The principal error source is the elevation antenna and its electronics.

Mean Glidepath Error (MGE) and PFN in the vertical degree of freedom vary differently with range. Separation is thus required in order to establish the displacement in feet which each term generates.

3.2.1 Mean Glidepath Error

From 3.2.1.12 of reference 6, the equipment is capable of MGE = 0.040° at the ARD. This angle (0.040°) must be invariant with distance. At the Cat I DH, the slant range is 3725' and the constant angle therefore implies

$$\text{MGE} = 3725\sin 0.04 = 2.6' \quad (3\text{-}8)$$

3.2.2 Path Following Noise

The performance of the equipment is not specified at the Cat I distance; it must therefore be extrapolated from the specification for Cat II; this extrapolation must be adjusted to allow for degradation of performance with distance. From 3.2.1.13 of reference 6, the angular value of the PFN capability of the equipment is 0.02° at the Cat II D' f 100' (967' from the ARD); this may degrade linearly with distance to 0.175° at 15NM (x_T = 91140'). An interpolation procedure yields the approximation PFN° = 0.020 + (0.175-0.020)[(x-967)/(91140-967)] where x is the aircraft distance from the ARD. This may be simplified to

$$PFN^\circ = 0.0183 + 1.720x/10^6 \tag{3-9}$$

From table A-1, the Cat II DH and the Cat I DH are at distances of x = 967 and x = 2872', respectively. Then, where the PFN at the Cat II distance is (845 + 967)tan0.020 = 0.63, integration yields the PFN at the Cat I DH as

$$PFN = 0.63 + \int_{x=967}^{x=2872} \{[0.0183+(1.7208x/10^6)]/57.3\}dx = 1.46' \tag{3-10}$$

3.2.3. Vertical Quantization Error

Vertical quantization is 0.1 meters. This has a corresponding error of $2[(0.1)/2\sqrt{3}\,]$= 0.06 meters = 0.2'.

3.2.4 Level Sensor Errors

The level sensors, from 3.7.9 of reference 6, have random errors of 0.05° in roll and 0.01° in pitch. The latter, producing an effect of $65\Delta\beta_P$, is subsumed within the MGE value given above. The former produces a random error of ($2.6\Delta\beta_R$) feet, as shown in (A-25-1); with $\Delta\beta_R = 0.05°$, this term contributes a random error of 0.13'.

3.2.5 Avionics

As in section 3.1.5, this error is 1.1'.

3.2.5 Total Vertical Error and Error Margin

The errors are combined RSS to yield the total system error of

$$\text{Total Error} = \sqrt{2.6^2 + 1.46^2 + 0.2^2 + 0.13^2 + 1.1^2} = 3.2' \tag{3-11}$$

This value is the total vertical error due to the various random sources.

As the total allowed vertical error is 15', the vertical error margin available for survey errors is therefore

$$\text{Error Margin} = 15 - 3.2 = 11.8. \qquad (3\text{-}12)$$

3.3 REQUIRED SETUP ACCURACY

The data are now in place to enable determining the accuracy required of the setup. The x-direction location of the Cat I DH window is not specified; the allowable error is thus unknown and is therefore entered below as (?). From (3-7) in 3.1.6, the margin allowed for the survey in the y-direction error for setup is 33.3'. From the end of section 3.2, (3-12) shows that the corresponding vertical error allowance is 11.8'. Finally, (A-25-1) and table A-5 present the errors of aircraft position in terms of the various elements. As the survey and setup error may not exceed the allowance, this yields, repeating (A-25-1) as (3-13),

Case 1

$$\begin{aligned} ? &= 0.998\Delta x_{DA} + 0.052\Delta z_{DA} - 0.052\Delta z_E \\ 33.3 &\geq \Delta y_{PA} + 3.4\Delta\alpha_R \\ 11.8 &\geq 0.052\Delta x_{DA} - 0.052\Delta x_E + \Delta z_E - 2.6\Delta\beta_R \end{aligned} \qquad (3\text{-}13)$$

where the survey position errors are in feet and the alignment errors are in degrees. The double subscript on Δx_{DA} is a reminder that the azimuth antenna and the DME are physically collocated. The double subscripted term Δy_{PA} is the error of estimated aircraft position due to the combined effects of the lateral survey errors of the azimuth antenna and its sighting pole, as discussed in subsection A.4 and (A-29).

In (3-13), there are two inequalities and seven unknowns; the first expression is useful as a "sanity check", but not otherwise for there is no x-direction accuracy requirement. This situation is mathematically under-determined, and more data or constraints, or other assumptions, are required to resolve the situation.

Table 3-1 presents a proposed budget for the adjustable elements of the setup, including the linear survey errors, and the angular alignment errors that are not automatically controlled. The linear allowances of the budget are presented in feetand in meters. The angular alignment budget elements are presented in degrees. It will be noticed that the proposed budget accuracies are not tight; it should be possible to set up the system in this Case 1 using a yard-stick or measuring tape instead of a survey theodolite. The proposed roll and pitch angular alignment accuracies are several times the accuracies (0.05°) of the several level sensors, reported above from references 5 and 6.

Under the extremely conservative assumption that the survey errors are such that their magnitudes add, we may evaluate (3-13) with various assumptions for the values of the allowable survey errors to find a set, presented in table 3-1, that satisfies the second two elements of (3-13). For example, the second line of (3-13) is $33.3 \geq \Delta y_{PA} + 3.4\Delta\alpha_R$.

allowable survey errors to find a set, presented in table 3-1, that satisfies the second two elements of (3-13). For example, the second line of (3-13) is $33.3 \geq \Delta y_{PA} + 3.4\Delta\alpha_R$.

Assume that the azimuth antenna roll budget is 0.2°; if the antenna is more accurately aligned, then there remains some budget for problems such as frost heaves. With this assumption, the second member of (3-13) is $33.3 \geq \Delta y_{PA} - 0.68$; this allows $32.6 \geq \Delta y_{PA}$. Using (A-29), with $\Delta y_{PA} = 32.6$ feet and $\rho_{DA} = 3725$ feet yields

$$|\Delta y_P| = |\Delta y_A| = [32.6 / (1 + 3725/500)] = 3.86' \qquad (3\text{-}14)$$

which is slightly more than 1 meter. The budget of 1 meter (3.3') is therefore proposed for the required accuracy of the survey for the azimuth antenna location and for the azimuth sighting pole location.

Table 3-1. Recommended Survey and Alignment Budget, Case 1

Linear Allowances, feet (meters)	Angular Alignment Allowances, degrees
$\Delta x_{DA} \leq 49.2$ (15 meters)	Azimuth Antenna Pitch $\Delta\alpha_P \leq 0.20$
$\Delta y_{DA} \leq 3.3$ (1 meter)	Azimuth Antenna Roll $\Delta\alpha_R \leq 0.20$
$\Delta y_P \leq 3.3$ (1 meter)	
$\Delta z_{DA} \leq 6.6$ (2 meters)	
$\Delta x_E \leq 49.2$ (15 meters)	
$\Delta y_E \leq 29.5$ (9 meters)	Elevation Antenna Roll $\Delta\beta_R \leq 0.20$
$\Delta z_E \leq 3.3$ (1 meter)	

Evaluating (3-13) with the entries of table 3-1 gives

$$\begin{aligned} ? &= 49.5 \\ 33.3 &> 27.9 \\ 11.8 &> 8.9 \end{aligned} \qquad (3\text{-}15)$$

so that the proposed allowances of the table satisfy the required conditions. In forming (3-15), it was necessary to invert (3-14) in order to find Δy_{PA}, given its components Δy_{DA} and Δy_P. The budget of table 3-1 presents allowances for errors of the survey, consistent with the allowable error margins. However, the coefficients for Δy_E and Δz_{DA}, developed in appendix A, are so small that no meaningful information on their accuracy requirements can be extracted; see (A-20-1). It is not realistic to leave their budgets unestablished, lest it be assumed that no

the comparable accuracy allowances, Δy_{DA} andΔz_E, respectively, so that their assigned allowances are $|\Delta y_E| \leq 29.5'$ and $|\Delta z_{DA}| \leq 6.6$.

Another view of the setup requirements, with a significant operational implication, is now considered. Return to (3-13), and now assume that the survey has been done exactly, so that the lateral offset of 150' is entered as $149.28' < y_D \leq 152.56'$ (45.5 meters $< y_D \leq 46.5$ m) and is therefore transmitted as $y_D = 46$ meters; also assume that the azimuth sighting pole has been exactly placed. In this case there is no survey error at all, and the only error in the lateral data entry is included in the quantization; that has already been accounted for in forming (3-13). Similarly assume that all the other survey elements have been correctly observed and entered. Then, in (3-13), all the linear entries are zero, i.e., $\Delta x_{DA} = \Delta y_{DA} = \Delta z_E = \Delta z_E = 0$, and (3-13) reduces to

$$
\begin{aligned}
33.3 &\geq -3.4\Delta\alpha_R \\
11.8 &\geq 2.6\Delta\beta_R
\end{aligned}
\qquad (3\text{-}16)
$$

The operational significance of (3-16) is considered. Solving (3-16) yields $\Delta\alpha_R \leq 9.8°$ and $\Delta\beta_R \leq 4.5°$. Then, if the survey has been done correctly and entered correctly, and the aircraft is on the runway centerline extension, the angular alignment could be off by as much as 9.8° in roll of the azimuth antenna, or 4.5° in roll of the elevation antenna without violating the specified conditions for this configuration. It is not proposed that the alignment should, or could, be that loosely controlled. Instead, the point is that it may, in tactical operations, be necessary to mount one or both of the antennas on icy or muddy ground, rather than on a pre-surveyed concrete pad. In this situation, if the initial survey and alignment have been correctly performed, the specified system accuracy will not be significantly degraded by events such as frost heaves or settling in the mud.

When the aircraft is far from the runway centerline extension, the misalignment of the antennas has greater effects. Assume the azimuth angle is 40°, as per 3.1.4.1.9.1.3.b of reference 1, so that the aircraft is far from the centerline; its lateral offset relative to the antennas is $2410\tan 40° = 1846'$. If a 3° glide slope is assumed, the slant range is 3749'. Using (A-24-1) and table A-5, the lateral error of position estimate due to roll misalignment of the azimuth antenna is $\Delta y = 3749\sin 3.4\Delta\alpha_R/57.3 = 3.4\Delta\alpha_R°$. Using (A-24-1) and table A-5, the vertical error of position estimate, due to roll of the elevation antenna, is $\Delta z = 3749\sin 40°\ \Delta\beta_R/57.3 = 42.0\Delta\beta_R$. When the elevation antenna is misaligned by 0.5°, the MMLS software shuts off the elevation guidance, and when the azimuth antenna is misaligned by 0.5° the MMLS software shuts off the entire system. At these limit values of misalignment, the misalignment error contributions due to roll of the antennas are 1.7' laterally, and 21' vertically, if the azimuth angle is 40°. The increment of lateral error due to the misalignment is negligible; the vertical is excessive. If approaches at such large-azimuth conditions are planned and required in some situation, then it is necessary to reduce the elevation antenna roll alignment budget to 0.05°, which reduces the error from 21' to 2.1'. However, if the maximum azimuth angle is 10°, then the vertical error is $11.3\Delta\beta_R$; when $\Delta\beta_R = 0.5°$ this yields 5.6'. In this case, restricting the

the vertical error is $11.3\Delta\beta_R$; when $\Delta\beta_R = 0.5°$ this yields 5.6'. In this case, restricting the elevation antenna roll in setup to the budget of 0.15° yields an error increment of 0.9', within the allowable margin of (3-15). Other elements of misalignment have second-order effects.

The budget proposed above is not the unique correct solution, for there is no unique correct solution. However, it is viewed as presenting a reasonable set of allowances for an under-determined case.

3.4 CASE 1 CONCLUSIONS

The magnitudes of the correlated survey and incremental angular alignment setup errors which may be tolerated in the assumed conditions are presented in table 3-1. The linear accuracies, stated in feet, are transmitted to the aircraft from the MMLS in meters; the equivalent survey error budget is provided parenthetically in meters. These linear accuracies appear to be within the capabilities of modern surveying equipment, even in a field procedure.

The assumption that the survey errors, and also especially the DME error, are always correlated in the disadvantageous direction, has special importance in understanding the implications of (3-13), where Δx_{DA}, Δy_{DA}, and Δz_E appear in two rows, each. Obviously, these quantities cannot be positive in one position in (3-13) and negative in the other; nonetheless, that impossibility has been assumed, so as to be very conservative.

The several members of (3-15) are not marginal. A lateral allowance of 3.3' (1 meter) is not critical. The third member of (3-15) has a relatively small margin for Δz_E, the z-direction component. This is due to the fact that the slant range is relatively large, and amplifies the various range-dependent errors such as the avionics. Even so, a setup in which an uncertainty, or error, of 3.3' in the height of the elevation antenna above the ground can be tolerated without violating the system specifications cannot be considered critical.

If the initial survey and alignment procedure is carried out correctly, and the aircraft is near the centerline, then relatively large alignment errors can be tolerated during operations without violating the specifications. But if the possiblity of large azimuth angles exists, then the elevation antenna roll alignment must be more accurately set up and maintained.

Finally, the sensitivity of the results to the random errors is considered. For the lateral errors, the ratio (Random/Allowable) = (R/W) = (33.3/65) = 0.5; from table A-6 and its discussion, a 10 percent change of the random errors induces a 10 percent change in the margin available for the setup budget. For the vertical errors, (R/W) = 0.21; a 10 percent change of the random errors induces a 2.7 percent change in the margin available for the setup. The inevitable uncertainties in the random data do not impinge seriously on the setup.

SECTION 4

CASE 2: SHORT-FIELD OPERATION

In some circumstances it might be required to site the elevation antenna as close as 200' toward the stop-end from the threshold, so that x_E = -200' instead of -845'. Such siting might be appropriate for a very short runway, or in some tactical operations. It is therefore assumed that the aircraft is on the runway centerline extension, the azimuth/DME combination is collocated with the elevation antenna at this forward location 150' from the runway centerline, and the glide slope is set at 4° so that the approaching aircraft is slightly higher at the threshold-crossing distance than if a nominal glide slope of 3° were used. This section examines the consequence of this siting. This site and that of Case I are very similar; Case I is therefore used as a format. Figure 4-1 shows the geometry.

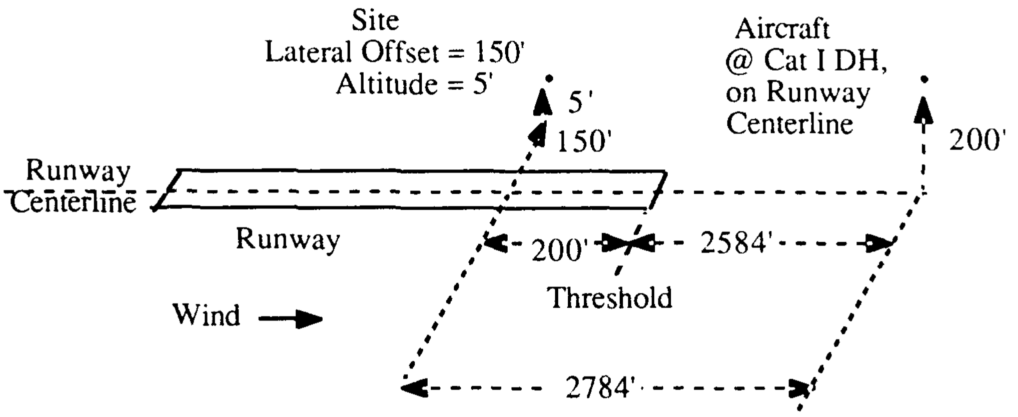

Figure 4-1. Geometry of Case 2

In figure 4-1, it should be noted that the x-distance from the equipment to the aircraft is much shorter than in Case 1. This is due to the higher glide slope used for this case. The threshold crossing altitude is 17.5'. The geometry is determined by the conditions that x_D = -200, y_D = 150', and glide slope = 4°.

4.1 LATERAL

The slant range in Case 2 is 2795', from table A-1. Then, as in (3-3) the lateral error due to the DME error is

$$\varepsilon_D = 150(644)/(2795 - 644) = 44.9' \qquad (4\text{-}1)$$

Following (3-4), the MCE is

$$MCE = 2795 \sin 0.0624 = 3.0' \tag{4-2}$$

Following (3-5), the PFN is

$$PFN = 2795 \sin 0.02 = 1.0' \tag{4-3}$$

The lateral quantization error is 1.9', the alignment error is $3.4\Delta\alpha_R = 0.2'$, as $\Delta\alpha_R = 0.05°$, and the avionics error is $2795\sin 0.017 = 0.8'$.

Combining these various effects, the lateral error due to random sources is

$$\sqrt{44.9^2 + 3.0^2 + 1.0^2 + 1.9^2 + 0.2^2 + 0.8^2} = 45.1' \tag{4-4}$$

This error is considerably greater than in Case 1, as the slant range is shorter due to the higher glide-slope; the reduced slant-range, and assumed negative range error exaggerate the DME error effect. The available survey error margin is

$$\text{Margin} = 65 - 45.1 = 19.9' \tag{4-5}$$

4.2 VERTICAL

Following (3-8), the MGE is

$$MGE = 2795 \sin 0.04° = 2.0' \tag{4-6}$$

Using the approach in section 3, expressed by the integration in (3-10), yields PFN of 1.1' in the geometry of Case 2.

The vertical quantizing error is 0.2', the random error due to the elevation antenna roll sensor is $2.62\Delta\beta_R = 0.13'$, and the avionics error is 0.8'.

Combining these various errors, the total random vertical error is

$$\sqrt{2.0^2 + 1.1^2 + 0.2^2 + 0.13^2 + 0.8^2} = 2.4 \tag{4-7}$$

and the vertical margin is

$$\text{Margin} = 15 - 2.4 = 12.6 \tag{4-8}$$

4.3 REQUIRED SETUP ACCURACY

Gathering the results developed above, and using (A-25-2),

$$
\begin{aligned}
? &= 0.995\Delta x_{DA} + 0.072\Delta z_{DA} - 0.070\Delta z_E \\
19.9 &\geq \Delta y_{PA} + 3.40\,\Delta\alpha_R \qquad (4\text{-}9) \\
12.6 &\geq 0.070\Delta x_{DA} - 0.070\Delta x_E + 0.995\Delta z_E - 2.62\Delta\beta_R
\end{aligned}
$$

As in section 3, assume the azimuth roll alignment errors are 0.2°. Then the second member of (4-9) reduces to $19.22 \geq \Delta y_{PA}$. Using (A-29), as in section 3, with $\rho = 2795'$, $\Delta y_P = \Delta y_A = 2.92'$; the value 2.90' will be used. This implies a relatively high, but not excessively difficult, level of precision in conducting the survey for the azimuth antenna and its sighting pole.

Evaluating (4-9) with the budget in table 4-1 yields

$$
\begin{aligned}
? &= 50.1 \\
19.9 &> 19.11 + 0.68 = 19.79 \qquad (4\text{-}11) \\
12.6 &> 10.7
\end{aligned}
$$

Table 4-1. Recommended Survey and Alignment Budget, Case 2

Linear Allowances, feet (meters)	Angular Alignment Allowances, degrees
$\Delta x_{DA} \leq 49.2$ (15 meters)	Azimuth Antenna Pitch $\Delta\alpha_P \leq 0.20$
$\Delta y_{DA} \leq 2.9$ (0.9 meters)	Azimuth Antenna Roll $\Delta\alpha_R \leq 0.20$
$\Delta y_P \leq 2.9$ (0.9 meters)	
$\Delta z_{DA} \leq 6.6$ (2 meters)	
$\Delta x_E \leq 49.2$ (15 meters)	
$\Delta y_E \leq 36.1$ (11 meters)	Elevation Antenna Roll $\Delta\beta_R \leq 0.20$
$\Delta z_E \leq 3.3$ (1 meter)	

4.4 CASE 2 CONCLUSIONS

Case 2 is the critical collocated case. This is due to the short-field condition, which mandates a higher glide slope and, because of the consequent short slant range and larger azimuth angle, exaggerates the lateral effect of the DME error. This reduces the available lateral margin. Nonetheless, the accuracies required for the survey and setup are well within the capabilities of modern survey equipments. The survey linear accuracies could perhaps be satisfied using yardsticks. The angular alignment accuracies are not severe; they are three times the nominal accuracy of the level sensors that are used to make the setup. This case is "critical" only in the sense that it leads to narrower ranges between the budget and the allowables; comparison with section 3 shows that the budget is satisfied more narrowly in the latter case than in the former. In a broader context, it is difficult to assert that a situation is "critical" when its setup allowances are as large as those proposed in table 4-2.

If the linear survey is done exactly, then, as in section 3, all of the allowance can be assigned to the alignment. This allows the azimuth antenna roll to be as much as $19.9/3.4 > 5.6°$, and the elevation antenna roll to be as much as $12.6/2.62 = 4.8°$. The lateral error increment is 1.9' at 0.5° of azimuth antenna roll, and 1.3' at 0.5° of elevation antenna roll. Both can be tolerated within the budget of table 4-2. But, as before, assume the aircraft is approaching on a 40° radial, or on a 10° radial. Then the errors due to misalignment are found as before. The lateral error increment is $3.4\Delta\alpha_R = 1.7'$ if the approach is 40° and $\Delta\alpha_R = 0.5°$; this is small enough to be tolerated, according to (4-11). If $\Delta\beta_R = 0.5°$, the vertical increment is $31.3\Delta\beta_R = 15.6'$ in the 40° approach case and is $8.5\Delta\beta_R = 4.2'$ in the 10° approach case. If the setup is maintained at 0.2° in elevation antenna roll and the approach radial is restricted to 10°, then the vertical increment is 0.8', an error increment which lies within the budget shown in (4-11) and table 4-2.

Referring to table A-6 and its discussion, the ratio (R/W) = 0.69 in lateral, and 0.16 in vertical. The corresponding sensitivities of the available margin to an uncertainty of 10 percent in the random errors are 22.7 percent in lateral and 1.9 percent in vertical. The lateral sensitivity is high, while the vertical sensitivity is low.

SECTION 5

CASE 3; SPLIT-SITE, 12,000 FOOT RUNWAY, CAT I PERFORMANCE

In Case 3, it is assumed that the elevation antenna is at the usual site, and that the DME and the azimuth antenna are collocated at the stop-end of the runway, 12000' from the ARD. In this situation, it is possible, and normal, for an aircraft to make a straight-in approach without use of the position reconstruction methods required when the azimuth antenna is offset from the runway centerline. Nevertheless, an automatic approach procedure can be used in this situation, and the analysis will be conducted under that assumption. There are two significant differences from the work in the preceding sections: the configuration is split, rather than collocated, and thus the geometry for the azimuth antenna/DME combination is different from that of the elevation antenna; further, the elevation antenna is offset 450' from the runway centerline. Consequently, there are different variables to be evaluated, but there will still be only two equations from which to perform this evaluation. The procedures used in section 3 will be used as a format and guide in this section, which, like section 4, is therefore relatively brief.

The assumed conditions which underlie this situation are:

a. Glide slope = 3°;
b. Aircraft at Cat I DH = 200', on runway centerline;
c. Elevation antenna has altitude = 5', with lateral offset of 450'.
d. Azimuth and DME antennas are collocated on the runway centerline, 12,000' from the ARD, at height of 5'.

With these conditions, the procedure of section 3 finds the elevation antenna to be located at $x_E = -731'$, and the aircraft is at $x_T = 2962'$, as shown in figure 5-1.

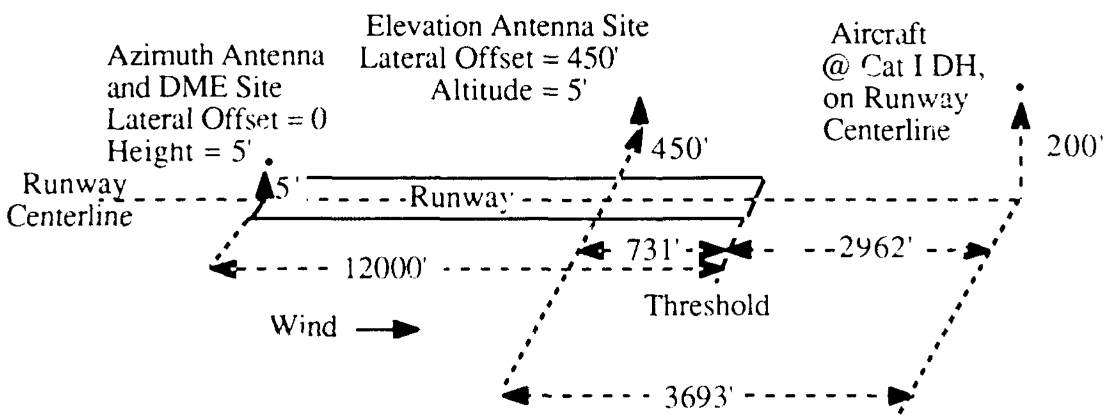

Figure 5-1. Geometry of Case 3

5.1 LATERAL

The error sources for lateral errors in this case are the azimuth antenna and the avionics. In principle, the DME is also an error source, but as the DME is on the runway centerline, its lateral error contribution is zero. The circumstances under which the DME is an actual error source will be discussed and shown to have a trivial effect.

From 3.2.1.13 of reference 5, the azimuth antenna equipment has accuracy of MCE = 0.06°. At the Cat I location of this case, the constant angle of MCE yields

$$MCE = (12000 + 2962)\sin 0.06 = 15.7' \qquad (5\text{-}1)$$

From 3.2.1.13 of reference 5, the equipment has angular PFN = 0.02°. The PFN error contribution is therefore

$$PFN = (12000 + 2962)\sin 0.02 = 5.2' \qquad (5\text{-}2)$$

From section 3.1.3, the survey lateral quantization error is 1.9'.

The azimuth roll effect is 3.4*0.05 = 0.17', and the avionics error of 0.017° at a slant range of 14963' results in a linear error of 14963sin0.017 = 4.4'.

Combining the MCE, PFN, and quantizing errors, RSS, the total random error in the lateral direction is $\sqrt{15.7^2 + 5.2^2 + 1.9^2 + 0.17^2 + 4.4^2} = 17.2'$. The total allowable is 65', as in section 3; the margin to be allowed for the survey is therefore 65 - 17.2 = 47.8'.

It may seem paradoxical that the effects of the equipment errors are smaller, and the margin available for the setup greater, when the azimuth and DME are almost two miles further from the ARD. This is altogether due to the fact that, in the present Case 3, the collocated azimuth antenna and DME are assumed to be on the runway centerline. Therefore, although the DME error is no less than in Case 1, its effect on the lateral position error is nil! The DME error does, of course, cause the indicated position of the aircraft to be more (or less) distant from its actual position. But this does not affect the lateral position error.

The assumption that both the aircraft and the azimuth antenna are located on the runway centerline obscures an interesting nonlinear coupling between the lateral survey errors and the random errors. For example, assume that the aircraft is on the centerline, but that the survey is in error so that the azimuth antenna is actually 40' off the centerline while the survey reports it to be on the centerline. As in Case 1, this causes the DME error to generate a lateral position error, if the aircraft is making a computed centerline approach. Note that this approach mode is not needed when the azimuth antenna is on the runway centerline; nonetheless, that mode might be used in some unforeseen circumstances. This error is easily found by using (3-2) and the approach of section 3.1.1, the error due to the DME is 644(40)/(12000 + 2963 - 644) = 1.7'; this is negligible compared to other errors, increasing the total RSS error from 17.2' to 17.3'.

5.2 VERTICAL

The aircraft is assumed to be at the Cat I DH, as in Case 1. In a split-site configuration, the elevation antenna can be up to 450' distant from the runway centerline; that offset is assumed as it establishes the most sensitive survey conditions. The procedure used here is the same as in section 3.2, except that the numerical data are slightly different. Distance to the Cat II DH is $\sqrt{(95/\tan 3)^2 - 450^2} = 1756'$, and the slant range from the elevation antenna to the Cat II location is $\sqrt{1756^2 + 450^2 + 95^2} = 1815'$.

From 3.2.1.12 of reference 6 and the data of table A-5, MGE = 3726sin0.04 = 2.6'.

Again following the procedure of section 3.2.2, and using the values in table A-1, PFN is, in degrees, PFN = $0.0182 + 1.72x/10^6$, and in feet

$$PFN' = 1815\tan 0.053 + \int_{x=1025}^{x=2962} \{[0.0182 + (1.72x/10^6)]/57.3\}dx = 2.4' \qquad (5\text{-}3)$$

The vertical quantizing error is 0.3'. The elevation antenna roll error contribution is 7.86*0.05 = 0.4'.

The avionics error is 1815sin0.017 = 0.5'.

The MGE, PFN, quantizing, and avionics errors are combined RSS to give total random error of $\sqrt{2.6^2 + 2.4^2 + 0.3^2 + 0.4^2 + 0.5^2} = 3.6'$. The survey margin is thus 15 - 3.6 = 11.4', a value similar to that deduced for Case 1.

5.3 REQUIRED SETUP ACCURACY

Gathering the results developed above, and equating these errors to (A-25-3),

Case 3

$$\begin{aligned} ? &\geq 0.999\Delta x_{DA} \\ 47.8 &\geq \Delta y_{PA} + 3.4\Delta\alpha_R \\ 11.4 &\geq 0.052\Delta x_{DA} - 0.052\Delta x_E + \Delta z_E + 7.86\Delta\beta_R \end{aligned} \qquad (5\text{-}4)$$

Table 5-1 shows a proposed budget of survey and alignment errors, appropriate for Case 3 Cat I operations. To the extent possible, the values which were acceptable in Case 1 are repeated here, for the same reasons. There are some differences. The criteria for the azimuth/DME siting survey accuracy may be relaxed slightly, because the azimuth antenna and the aircraft are presumably on the runway centerline, and some of the sensitivity coefficients are consequently reduced.

Table 5-1. Recommended Survey and Alignment Budget, Case 3

Linear Allowances, feet (meters)	Angular Alignment Allowances, degrees	
$\Delta x_{DA} \leq$ 49.2 (15 meters)	Azimuth Antenna Pitch	$\Delta\alpha_P \leq 0.20$
$\Delta y_{DA} \leq$ 1.48' (0.45 meters)	Azimuth Antenna Roll	$\Delta\alpha_R \leq 0.20$
$\Delta y_P \leq$ 1.48' (0.45 meters)		
$\Delta z_{DA} \leq$ 6.6 (2 meters)		
$\Delta x_E \leq$ 49.2 (15 meters)		
$\Delta y_E \leq$ 45.9 (14 meters)	Elevation Antenna Roll	$\Delta\beta_R \leq 0.20$
$\Delta z_E \leq$ 3.3 (1 meter)		

With the values in table 5-1, the error margins are

$$\begin{aligned} ? &= 49.2' \\ 47.8 &= 47.8' \\ 11.4 &> 10.0' \end{aligned} \tag{5-5}$$

Inspection of (5-5) shows that the required angular alignment budgets can be increased, if desired. However, this is not needed, for the alignment budgets are adequate. The lateral survey accuracy of 1.48' for the locations of the azimuth antenna and the azimuth sighting pole is not severe, since they are both on the runway centerline. Assume a tactical situation, wherein the MMLS may be set up on a section of highway: it is not difficult to locate the center of a highway to the accuracy of 1.48' by pacing.

Assume that the survey is conducted correctly. Then the entire allowance can be reserved for the alignment; this yields 47.8/3.40 = 14° in azimuth antenna roll, or 11.4/7.86 = 1.4° of elevation antenna roll.

Alternately, assume that the aircraft is on a 40° or a 10° radial approach to the threshold, and is on the opposite side of the runway from the elevation antenna site. These appraoches imply lateral offsets of the aircraft from the runway centerline of 2485' and 522', respectively, and the corresponding lateral offsets relative to the elevation antenna are 2935' and 972', with planar azimuth angles at the elevation antenna of 44.7° and 18.2° and slant ranges of 4174' and 3129'. The elevation of the aircraft with respect to the azimuth antenna is 0.747° for both geometries. Using (A-24), the alignment error contributions become $51.2\Delta\beta_R$ and $17.2\Delta\beta_R$ vertically, for the two approach radials, respectively and $0.34\Delta\alpha_R$ for the lateral error for both approach geometries. As before, these results are scaled in degrees. If $\Delta\alpha_R = \Delta\beta_R = 0.5°$, the lateral increment of error is 0.17' for both approach radials; this is negligible. The

increment of vertical error is 25.6' and 8.6' for the two approach radials; these are large enough that they may not be disregarded; the approach radials must be restricted, or the large lateral offset of the elevation antenna must be reduced to about 150', or the roll alignment of the elevation antenna must be maintained to the limit of the equipment capability. If the elevation antenna roll is maintained to 0.05°, then the vertical error contribution reduces to 2.6' and 0.9' for the two approach radials. Another alternate is to place a way-point on the runway centerline at or downwind from the distance corresponding to the Cat I DH; this reduces the azimuth angle which the aircraft subtends relative to the elevation antenna, and thus reduces the coefficients of the sensitivity of vertical error to elevation antenna roll.

5.4 CASE 3 CONCLUSIONS

The combined alignment and survey errors which can be tolerated in Case 3, under the stated assumptions, are given in table 5-1. The required accuracies are within the capability of modern surveying equipment. If the survey is performed correctly, the azimuth antenna roll can be as large as 3.5° without violating the established specification. The ratio of random-to-allowable errors in this case is (R/W) = 17.2/65 = 0.26 for lateral and 3.6/15 = 0.24 in vertical, so that 10 percent uncertainties in the random data imply 3.6 percent and 3.2 percent changes in the margin available for the setup.

SECTION 6

CASE 4; SPLIT-SITE, 12,000 FOOT RUNWAY, CAT II PERFORMANCE

In this case, the elevation antenna is sited exactly as in Case 3 and the azimuth antenna/DME combination is sited on the centerline 12000' from the ARD. The aircraft is assumed to be on the runway centerline, on a 3° glide slope, at the Cat II DH of 100'; see figure 6-1. The abbreviated format used in sections 4 and 5 will be used again.

The conditions which underlie the situation in Case 4 are restated:

a. Glide slope = 3°;
b. DH = 100', for Cat II;
c. Elevation antenna y_E = 450' as in Case 3;
d. Collocated azimuth antenna/DME located on centerline 12,000' from the ARD;
e. Height of all units above ground = 5';
f. From table A-1, the Cat II distance is 1025'.

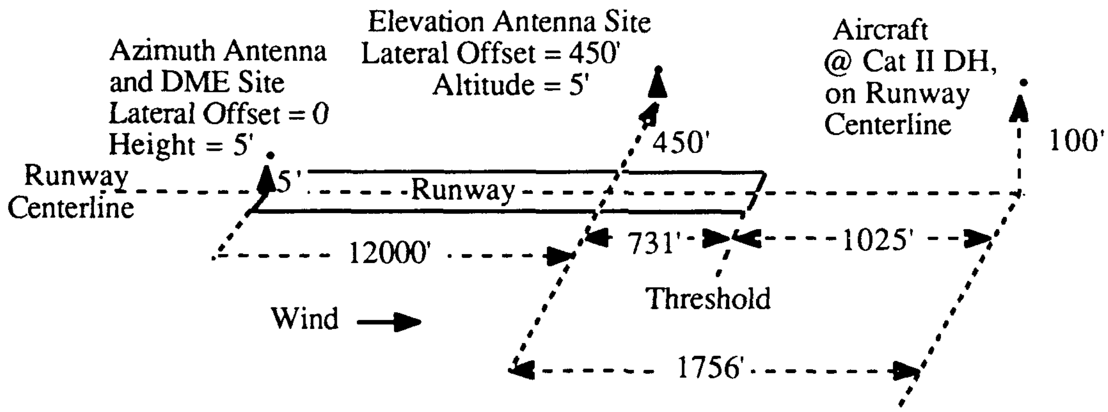

Figure 6-1. Geometry of Case 4

6.1 LATERAL

The random contributors to lateral error are MCE, PFN, quantization, and the avionics. Except for the nonlinear coupling of survey errors and DME random errors considered and disposed of in section 5, the DME errors do not contribute. As before, it is necessary to separate the contributions of MCE and PFN, as they vary with range in different ways.

From 3.2.1.13 of reference 5, the MCE is 0.06°. At the Cat II location the MCE is therefore

$$MCE = (12000 + 1025)\sin 0.06 = 13.6' \qquad (6\text{-}1)$$

From 3.2.1.13 of reference 5, PFN = 0.02°, which yields, at the Cat II DH,

$$PFN = (12000 + 1025)\sin 0.02 = 4.5' \qquad (6\text{-}2)$$

From section 3.1.3, the quantizing error is 1.9'. Using (A-25-4), the random effect of the azimuth antenna roll sensor error is $1.66\Delta\alpha_R = 0.08'$, as $\Delta\alpha_R = 0.05°$.

The lateral avionics error is $13025\sin 0.017 = 3.9'$.

Combining RSS the contributions of MCE, PFN, and quantization yields the total random error as $\sqrt{13.6^2 + 4.5^2 + 1.9^2 + 0.08^2 + 3.9^2} = 15.0'$. The total lateral error allowed for Cat II is 30'. The margin for survey and setup errors is thus

$$\text{Margin} = 30 - 15 = 15' \qquad (6\text{-}3)$$

6.2 VERTICAL

The elevation antenna and the avionics are the contributors to the vertical error. The contributions of MGE and PFN are determined separately, as before, for the Cat II distance.

From 3.2.1.12 of reference 6, MGE = 0.04°. The slant range is 1815', from table A-5, and thus

$$MGE = 1815\sin 0.04 = 1.3' \qquad (6\text{-}4)$$

From 3.2.1.12. of reference 6, elevation PFN is 0.02°. Therefore the angular value of PFN implies a linear error of PFN = 1815sin 0.04 = 0.6'.

From section 3.2.3, the quantization error is 0.3'. The random effect of the elevation antenna roll is $7.86*\Delta\beta_R = 0.4'$. The avionics vertical error is $1815\sin 0.017 = 0.5'$.

Combining these results RSS the total random error in the vertical direction is $\sqrt{1.3^2 + 0.6^2 + 0.3^2 + 0.4^2 + 0.5^2} = 1.55'$. The margin for setup and survey is thus

$$\text{Error Margin} = 6 - 1.55 = 4.45' \qquad (6\text{-}5)$$

6.3 REQUIRED SETUP ACCURACY

The error margins available for the setup are gathered from the results above and are equated to the right of (A-25-4). the findings are presented as (6-6).

Case 4

$$\begin{aligned} ? &= \Delta x_{DA} \\ 15.0 &\geq \Delta y_{PA} + 1.66\Delta\alpha_R \\ 4.45 &\geq 0.051\Delta x_{DA} - 0.051\Delta x_E + \Delta z_E + 7.86\Delta\beta_R \end{aligned} \tag{6-6}$$

In lateral, assume a roll allowance of the azimuth antenna of 0.2°; then 14.68' can be allowed for Δy_{PA}. With slant range of 13026', the result is

$$\Delta y_{DA} = \Delta y_P = 0.54' \tag{6-7}$$

Again, as in the prior sections, this requires much more precision in the survey, but the accuracy required is not beyond the capability of a trained crew with simple equipment such as a measuring tape, a string, or the monitoring cable as a measuring tape.

The errors which can be tolerated in the vertical direction are smaller and require some discussion and explanation before presenting a budget. Assume a budget of 9.8' (3 meters) for Δx_{DA} and Δx_E; these are much tighter than in the prior three cases, although not difficult to establish in a field tactical procedure. Then (6-6) reduce to

$$\begin{aligned} ? &= 9.88 \\ 15 &= 15.0 \\ 4.45 &\geq 0.051(9.8) + 0.051(9.8) + \Delta z_E + 7.86\Delta\beta_R \end{aligned} \tag{6-8}$$

Evaluating the third member of (6-8) yields $4.45 \geq 0.5 + 0.5 + \Delta z_E + 7.86\Delta\beta_R$, yielding

$$3.45 \geq \Delta z_E + 7.86\Delta\beta_R \tag{6-9}$$

The budget of $\Delta\beta_R \leq 0.15°$ is proposed; this allows a survey error budget of $\Delta z_E \leq 3.45 - 7.86(0.15) = 2.27'$ or 0.7 meters.

These values are gathered in table 6-1, which presents the budget for Case 4. These values yield (6-11) when inserted in (6-6):

$$\begin{aligned} ? &= 9.8 \\ 15.0 &= 15.0 \\ 4.45 &> 4.40 \end{aligned} \tag{6-11}$$

Table 6-1. Recommended Survey and Alignment Budget, Case 4

Linear Allowances, feet (meters)	Angular Alignment Allowances, degrees
$\Delta x_{DA} \leq$ 9.8 (3 meters)	Azimuth Antenna Pitch $\Delta\alpha_P \leq 0.20$
$\Delta y_{DA} \leq$ 0.54 (0.16 meters)	Azimuth Antenna Roll $\Delta\alpha_R \leq 0.20$
$\Delta y_P \leq$ 0.54 (0.16 meters)	
$\Delta z_{DA} \leq$ 6.6 (2 meters)	
$\Delta x_E \leq$ 9.8 (3 meters)	
$\Delta y_E \leq$ 9.8 (3 meters)	Elevation Antenna Roll $\Delta\beta_R \leq 0.15$
$\Delta z_E \leq$ 2.27 (0.6 meter)	

6.4 CASE 4 CONCLUSIONS

The setup and survey errors which can be tolerated in Case 4 are presented in table 6-1. This case is critical because it is a Cat II setup and the allowable errors are uniformly smaller. The setup accuracies which are required for this case are considered to be within the capability of the setup crew, using modern survey instruments. However, the requirements are so loose that it might be possible to achieve a satisfactory setup using yardsticks, pacing, or the monitor cables for the linear displacements, and the built-in angular inclinometers for the alignment.

If the survey is perfectly conducted, the entire allowance may be used for the alignment; this yields 15.0/1.66 = 9° for azimuth antenna roll and 0.5° for roll of the elevation antenna. Offset radial approaches cannot be tolerated in a Cat II situation, for the vertical error situation allows no added errors. Again, this is not an absolute prohibition, for if a way point is set at the Cat I location, then the aircraft is close enough to the centerline at the Cat II distance for the alllowances to suffice. Alternately, as before, the sensitivity of the situation is significantly reduced if the elevation antenna is 150' from the runway centerline instead of 450' as in the present assumption.

The ratio of random error to the window is (R/W) = 15/30 = 0.5 in lateral and 1.5/6 = 0.25 in vertical, so that 10 percent changes in the random data imply changes in the available margin of 10 percent and 3.3 percent, respectively.

SECTION 7

RESULTS AND DISCUSSION

The various results, from sections 3 through 6, are gathered in table 7-1.

The required accuracies of the survey are feasible and impose no special needs on the survey and alignment equipments and procedures. The requirements of the several cases are discussed below.

There are two collocated configuration cases, Cases 1 and 2. Case 2 is more critical than Case 1. This is due to the fact that the short-field situation in Case 2 mandates a higher glide slope than in Case 1. As a result, the slant range is decreased, the azimuth angle is increased, and therefore the lateral error due to the DME error is increased. Therefore, Case 2 has a much narrower lateral survey budget of 16.4' (5 meters). In this context, compare (3-14) and (4-11). Compare also the lateral margins for Cases 1 and 2, shown in table 7-1.

Cases 3 and 4 are split-site cases; Case 3 requires Cat I performance, while Case 4 requires Cat II performance. In table 7-1, the vertical margin for this case is relatively small, and dominates the tighter requirements for the survey and alignment accuracy requirements. It is thus entirely to be expected that Case 4 is the critical member of this pair with respect to the allowances for the survey. Compare (5-5) and (6-7); in the latter situation, the remaining lateral and vertical margins are much smaller than in the former.

In several cases, the budget for angular alignment can be relaxed; this has been noted at the various points where it is possible. However, if the survey and siting are carried out more accurately than the requirements of the budget, the angular alignment requirements become significantly looser. Specifically, if the survey accuracy is such that the equipments' sites are encoded accurately within the quantization interval, then the angular alignments may be allowed to degrade significantly; the limits of permissable degradation are shown in table 7-2. These allowances may enable the equipments to operate acceptably through disturbances such as frost heaves, or settling of muddy foundations, in a tactical situation.

A general observation may be drawn from the results shown in tables 7-1 and 7-2. The x-direction and y-direction survey accuracy requirements are so loose that they can successfully be met without survey instruments; pacing, or use of the monitor connector cables as a measuring device, should suffice, and should produce the required accuracy for the horizontal survey of the sites for the azimuth and elevation antennas. For measuring the vertical emplacement of the antennas, a simple optical device such as a levelled gunsight mounted on the elevation antenna should be adequate if pacing and visual estimation are not acceptable.

Offset radial approaches, rather than along the runway centerline, impose a severe burden on the roll alignment of the elevation antenna. It may be desirable to limit the approach radial to be with 10° of the runway centerline, to emplace the elevation antenna as close as possible to the runway, or to use a way point on the runway centerline extension at or downwind of the Cat I location so that the aircraft will be on or near the runway centerline when it is at the Cat I or Cat II decision heights.

TABLE 7-1. Summary Of Conditions And Allowable Setup Errors

	Case 1 Cat I Collocated			Case 2 Cat I Coll., Short Field			Case 3 Cat I Split			Case 4 Cat II Split		
Conditions												
	x	y	z	x	y	z	x	y	z	x	y	z
Locations												
DME/Az.	-845	150	5	-200	150	5	-12000	0	5	-12000	0	5
El.	-845	150	5	-200	150	5	-731	450	5	-731	450	5
Aircraft	2872	0	200	2584	0	200	2962	0	200	1025	0	100
Survey Linear Allowances, feet (meters), and Margin												
DME/Azimuth Antenna												
Δx_{DA}	49.2 (15)			49.2 (15)			49.2 (15)			9.8 (3)		
Δy_{DA}	3.3 (1)			2.9 (.9)			1.5 (.4)			0.5 (.16)		
Δy_P	3.3 (1)			2.9 (.9)			1.5 (.4)			0.5 (.16)		
Δz_{DA}	6.6 (2)			6.6 (2)			6.6 (2)			6.6 (2)		
Lateral Margin	33.3			19.9			47.8			15.0		
Elevation Antenna												
Δx_E	49.2 (15)			49.2 (15)			49.2 (15)			9.8 (3)		
Δy_E	29.5 (9)			36.1 (11)			45.9 (14)			9.8 (3)		
Δz_E	3.3 (1)			3.3 (1)			3.3 (1)			2.3 (0.7)		
Vertical Margin	11.8			12.6			11.4			4.45		
Angular Alignment Allowances, degrees												
$\Delta\alpha_P$	0.20			0.20			0.20			0.20		
$\Delta\alpha_R$	0.20			0.20			0.20			0.20		
$\Delta\beta_R$	0.20			0.20			0.20			0.15		

Table 7-2. Allowable Angular Alignment Degradation

	Case 1	Case 2	Case 3	Case 4
$\Delta\alpha_R$	9.8°	5.6°	14.0°	9.0°
$\Delta\beta_R$	4.5	4.8	1.4	0.5

It has been assumed that the emplacement of the azimuth antenna and of the azimuth sighting pole are conducted by independent survey procedures. This is not necessarily the case. Assume, for example, that a site is selected for the azimuth antenna, and that the location of the sighting pole is determined by using a transit or theodolite positioned at the selected azimuth site. Then the error of the azimuth yaw alignment, which totally dominates the determination of the budgets for Δy_{DA} and Δy_P, almost completely vanishes, and the allowance for Δy_{PA} may be used for the budget of Δy_A. Further, it has been assumed that the azimuth sighting pole is 500' from the azimuth antenna. If that distance is doubled, then the budgets for Δy_{DA} and Δy_P are also approximately doubled, as a consequence of (A-29).

SECTION 8

CONCLUSIONS

It is possible to emplace the MMLS ground units to an accuracy sufficient for meeting Cat I and Cat II operations in the several scenarios examined without use of surveying equipment. The survey data which must be encoded in the equipment can be successfully determined in the horizontal plane (x and y directions) by pacing or by use of the monitor connector cables as a measuring tape. In the vertical direction, the only stringent criterion is the altitude of the elevation antenna, where an error of 2.3' can be allowed in the most difficult situation of Case 4, a Cat II approach. An accuracy of 2.3' in the vertical direction is not difficult. The angular alignment requirements for the azimuth antenna are loose, and the antenna may be left unattended, even if the terrain is soft or subject to changes such as in frost heaves.

However, if aircraft may approach at large azimuth angles, with large values of off-center radials, then the allowances and budgets in the vertical direction, where the elevation antenna roll alignment becomes a large error contributor, must be more carefully controlled. This is particularly important in Cat II operations. Several means of reducing this limitation are suggested. Limiting the lateral offset of the elevation antenna to the minimum has the effect of reducing the sensitivity of the vertical error to roll of the elevation antenna. Alternately, using a way point on the runway centerline extension, at about one mile from the threshold, so that the aircraft is approximately on the runway centerline at the Cat I or Cat II points, would effectively eliminate the problem.

LIST OF REFERENCES

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2. Radio Technical Commission for Aeronautics, July, 1981, *Minimum Operational Performance Standards for Microwave Landing System (MLS) Airborne Receiving Equipment*, RTCA/DO-177, Washington, DC.

3. Radio Technical Commission for Aeronautics, July, 1981, *Minimum Operational Performance Standards for Airborne MLS Area Navigation Equipment*, RTCA/DO-198, Washington, DC.

4. Collins Radio Co.,8 Sept., 1974, *System Segment Specification for TACAN Navigation Set AN/ARN-118(V)*, Cedar Rapids, Iowa.

5. Bell Aerospace Corp., November, 1988, *Prime Item Development Specification for the Azimuth Antenna Assembly*, CDRL Item No. 106, Report No. 6531-947001, Niagara Falls, NY.

6. Bell Aerospace Corp., November, 1988, *Prime Item Development Specification for the Elevation Antenna Assembly,* CDRL Item No. 106, Report No. 6531-947002, Niagara Falls, NY.

APPENDIX A

SENSITIVITY OF AIRCRAFT POSITION ESTIMATE TO ACCURACY OF GROUND UNIT LOCATION DATA

The Microwave Landing System (MLS) is in principle capable of supporting area navigation (RNAV), and final approach and landing operations. In these procedures, the ground units transmit to the aircraft various data words which define key characteristics of the ground units. These data words are of two types: Basic and Auxiliary. Basic Words advise of the status of the several ground units, and the approximate locations of the ground units, etc. The Auxiliary Words provide the locations of the ground units to much more precise values. The Basic Words are essential and are sufficient if the aircraft plans only to make straight-in approaches. But if the aircraft is to undertake RNAV, or computed centerline approaches, the Auxiliary Words are also needed. In addition to these words, the ground units transmit signals from which the aircraft can deduce azimuth, distance, and elevation; these observations are combined with the various words to enable RNAV operations and computed centerline approach.

This appendix examines the following question: What is the sensitivity of the accuracy with which the aircraft position is estimated to the accuracy of the survey that provides the positions of the three ground units and to the accuracy of the angular alignment of the azimuth and elevation antennas? Three specific topics form the organizing principle: the theoretical structure is defined, the numerical results are generated, and the sensitivity of the results and conclusions to the asssumptions and data is considered.

A.1 THEORY

The notation is repeated for convenience. Figure A-1 shows a general configuration of the equipments, and defines the various coordinates and observations.

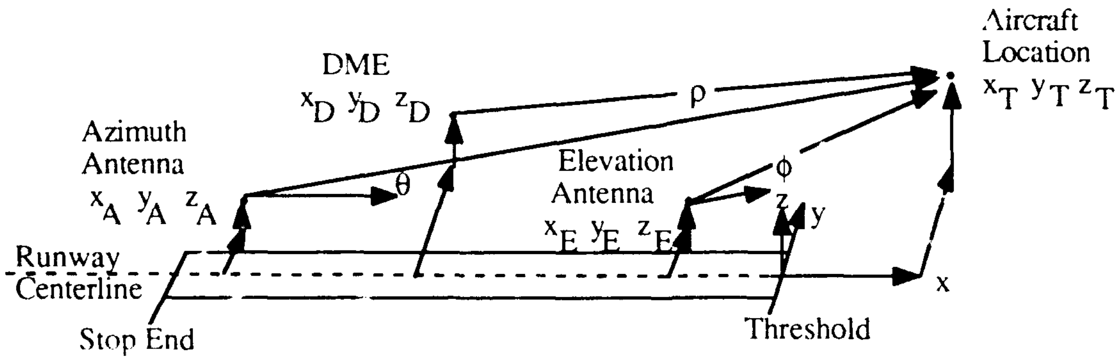

Figure A-1. System Installation Layout

x, y, z	Components of estimated position of aircraft
x_A, y_A, z_A	Components of true location of azimuth antenna
x_D, y_D, z_D	Components of true position of DME
x_E, y_E, z_E	Components of true position of elevation antenna
x_T, y_T, z_T	Components of true position of the aircraft
x_S, y_S, z_S	Components of the errors of the survey
θ	Observed azimuth angle
ϕ	Observed elevation angle
ρ	Observed slant range from the DME transmitter
ρ_E	Slant range from elevation antenna to aircraft
θ_B	Boresight-angle of azimuth antenna, assumed zero
$\Delta\theta$	Azimuth antenna alignment error
$\Delta\phi$	Elevation antenna alignment error
$\alpha_Y, \alpha_P, \alpha_R$	Azimuth antenna yaw, pitch, roll alignment
$\beta_Y, \beta_P, \beta_R$	Elevation antenna yaw, pitch, roll alignment

Assumptions

1. The runway is without tilt in any direction, and is flat and straight;
2. *The azimuth antenna is of conical pattern*;
3. The aircraft is on or very near the runway centerline, at the Cat I decision height (DH) of 200' or Cat II DH of 100', on an appropriate glide slope;
4. The nominal threshold-crossing is at the ARD, with height of 50';
5. The antenna phase centers are assumed to be 5' above the runway;
6. The computed centerline approach mode is used and the position reconstruction algorithm is free of errors.

Mathematics

The observations at the aircraft are defined by:

Observed distance from DME

$$\rho = \sqrt{(x_T-x_D)^2 + (y_T-y_D)^2 + (z_T-z_D)^2} \qquad \text{(A-1)}$$

Observed azimuth angle

$$\tan\theta = -(y_T-y_A)/\sqrt{(x_T-x_A)^2 + (z_T-z_A)^2} \qquad \text{(A-2)}$$

Observed elevation angle

$$\tan\phi = (z_T-z_E)/\sqrt{(x_T-x_E)^2 + (y_T-y_E)^2} \qquad \text{(A-3)}$$

The observations are in terms of the true positions of the ground units; (A-2) and (A-3) define cones in space.

Problem Restatement

The question, stated above in philosophical terms, is now restated. If the angular alignments of the antennas, or the transmitted geometrical site data (x_D, y_D, ..., z_E), are not exactly correct, what errors of calculated aircraft position result, and thence, what errors in these setup data may be accepted for various types of operation? It seems obvious that the most sensitive situation is during final approach, rather than during RNAV, and this study is therefore carried out for that case; specifically, the aircraft is assumed to be on or very near the centerline of the desired runway at the Cat I and Cat II decision heights and distances, while the ground units are arranged in collocated and split-site configurations.

Mathematical Problem Statement

Where the observed data are given exactly by (A-1)-(A-3), determine the effects on the estimated position (Δx, Δy, and Δz) due to survey errors of Δx_D, Δy_D, Δz_D, Δx_A, Δy_A, Δz_A, Δx_E, Δy_E, Δz_E and due to the antennas' alignment errors.

This problem statement leads to the generalized error expressions

$$x_T+\Delta x = (x_D+\Delta x_D)+\sqrt{\rho^2-(y_T+\Delta y-y_D-\Delta y_D)^2-(z_T+\Delta z-z_D-\Delta z_D)^2} \qquad \text{(A-4)}$$

$$y_T+\Delta y = (y_A+\Delta y_A)-\sqrt{(x_T+\Delta x-x_A-\Delta x_A)^2+(z_T+\Delta z-z_A-\Delta z_A)^2}\,\tan(\theta+\Delta\theta) \qquad \text{(A-5)}$$

$$z_T+\Delta z = (z_E+\Delta z_E)+\sqrt{(x_T+\Delta x-x_E-\Delta x_E)^2+(y_T+\Delta y-y_E-\Delta y_E)^2}\,\tan(\phi+\Delta\phi) \qquad \text{(A-6)}$$

The ideal procedure for evaluating the increments is to use truncated Taylor series, which yield

$$\Delta x = x'_{xD}\,\Delta x_D + x'_{yD}\,\Delta y_D + .. + x'_{yA}\,\Delta y_A + .. + x'_{zE}\,\Delta z_E + ... \qquad \text{(A-7)}$$

$$\Delta y = y'_{xD}\,\Delta x_D + y'_{yD}\,\Delta y_D + .. + y'_{yA}\,\Delta y_A + .. + y'_{zE}\,\Delta z_E + ... \qquad \text{(A-8)}$$

$$\Delta z = z'_{xD}\,\Delta x_D + z'_{yD}\,\Delta y_D + .. + z'_{yA}\,\Delta y_A + .. + z'_{zE}\,\Delta z_E + ... \qquad \text{(A-9)}$$

where x'_{xD} is the partial derivative of x with respect to x_D, etc. These partial derivatives are to be evaluated at x_T, y_T, z_T, x_D, y_D, ..., y_E, and z_E.

The implications of (A-7)-(A-9) are:

There are three dependent variables, Δx, Δy, and Δz;
There are eleven independent variables, Δx_D, Δy_D, ..., Δy_A, ..., Δz_E, plus $\Delta\theta$ and $\Delta\phi$, which are included as they are functions of the alignment errors.

Therefore there are 33 partial derivatives to be determined.

The transpose of the vector of increments of the independent variables is

$$\underline{e}^T = (\Delta x_D, \Delta y_D, \Delta z_D, \Delta x_A, \Delta y_A, \Delta z_A, \Delta x_E, \Delta y_E, \Delta z_E, \Delta\theta, \Delta\phi)^T \tag{A-10}$$

where the superscript T implies transpose. Similarly, the transposed vector of the resulting aircraft location estimate errors is

$$\underline{\varepsilon}^T = (\Delta x, \Delta y, \Delta z) \tag{A-11}$$

Partially differentiating (A-1)-(A-3) as indicated in (A-7)-(A-9) yields the matrix equation

$$M\underline{\varepsilon} = B\underline{e} \tag{A-12}$$

where the vectors $\underline{\varepsilon}$ and $\underline{e}$ have been defined, with solution

$$\underline{\varepsilon} = M^{-1}B\underline{e} = S\underline{e} \tag{A-13}$$

where S is the sensitivity coefficient matrix. The matrix M is

$$M = \left\{ \begin{array}{ccc} 1 & (y-y_D)/R_D & (z-z_D)/R_D \\ (x-x_A)\tan\theta/R_A & 1 & (z-z_A)\tan\theta/R_A \\ -(x-x_E)\tan\phi/R_E & -(y-y_E)\tan\phi/R_E & 1 \end{array} \right\} \tag{A-14}$$

The transpose of the matrix B, neglecting for the moment the alignment terms, is

$$B^T = \left\{ \begin{array}{ccc} 1 & 0 & 0 \\ (y-y_D)/R_D & 0 & 0 \\ (z-z_D)/R_D & 0 & 0 \\ 0 & (x-x_A)/R_A & 0 \\ 0 & 1 & 0 \\ 0 & (z-z_A)/R_A & 0 \\ 0 & 0 & -(x-x_E)/R_E \\ 0 & 0 & -(y-y_E)/R_E \\ 0 & 0 & 1 \end{array} \right\} \tag{A-15}$$

where R_A and R_E are the radii of the azimuth and elevation cones, and

$$R_D = \sqrt{\rho^2 - (y-y_D)^2 - (z-z_D)^2} \tag{A-16}$$

$$R_A = \sqrt{(x-x_A)^2 + (z-z_A)^2} \tag{A-17}$$

$$R_E = \sqrt{(x-x_E)^2 + (y-y_E)^2} \tag{A-18}$$

It is not illuminating to write the inverse of M in analytical form, and the analysis now turns to numerical results.

A.2 NUMERICAL RESULTS

Table A-1 presents the data which define the assumed locations of the ground units and the aircraft, and the observations appropriate to those situations. Distances, calculated as shown below for Case 1, are relative to the threshold.

Table A-1. Configuration Data

Conditions	Case 1 Cat I Collocated x	y	z	Case 2 * Cat I Collocated x	y	z	Case 3 Cat I Split x	y	z	Case 4 Cat II Split x	y	z
Locations												
DME	-845	150	5	-200	150	5	-12,000	0	5	-12,000	0	5
Az. Anten.	-845	150	5	-200	150	5	-12,000	0	5	-12,000	0	5
El. Anten.	-845	150	5	-200	150	5	-731	450	5	-731	450	5
Aircraft	2872	0	200	2584	0	200	2962	0	200	1025	0	100
Observations												
ρ_{DA} feet	3725			2795			14,963			13,025		
θ degs.	2.307			3.074			0.0			0.0		
ϕ degs.	3.000			4.000			3.000			3.000		
Geometry												
R_D feet	3717			2784			14,962			13,025		
R_A	3722			2791			14,964			13,025		
R_E	3721			2788			3720			1813		
ρ_E	3725			2795			3720			1815		
x @ Cat II DH	967			1150			1025			1025		

* The glide slope for Case 2 is 4°; the glide slope for the other cases is 3°.

Case 1 example. The glide slope is 3°, the aircraft height above the runway at the ARD is 50', and the elevation antenna height above the ground is 5'. The lateral offset of the collocated ground units is 150' so that from (A-3), by setting $x_T = 0$, we have $x_D = x_A = x_E = -\sqrt{(45\cot 3)^2 - 150^2} = -845'$. At the Cat 1 DH, $z_T = 200'$, and thus, using (A-1), $x_T = -845 + \sqrt{(195\cot 3)^2 - 150^2} = -845 + 3718'$, and thus $x_T = 2872'$. Slant range from

the azimuth antenna and the DME, ρ_{DA}, is therefore $\sqrt{3717^2+150^2+195^2} = 3726'$, and in this case the slant range from the aircraft to the elevation antenna, ρ_{DE}, equals the slant range to the DME because of the assumption of collocation in this case. The azimuth angle is $\arctan[150\sqrt{3717^2+195^2}] = 2.307°$, while the elevation angle is exactly 3°. The same procedure is used for the other cases. The terms R_D, R_A, and R_E are calculated from (A-16) through (A-18), using the geometry data generated and presented in the upper part of table A-1. The locations of the equipment and the aircraft in the upper part of table A-1 are with respect to the ARD. Note that the locations of the azimuth and DME units must be identical, as these two units are combined in the MMLS.

Table A-2 presents the elements of the matrices M for these cases, and table A-3 presents the elements of the corresponding matrices M^{-1}. In tables A-2 and A-3, some of the entries are integers, with values of 1 or zero rather than 0.9999 or 0.0000.

Table A-2. Matrices M

Case 1			Case 2			Case 3			Case 4		
1	-0.040	0.052	1	-0.054	0.070	1	0	0.013	1	0	0.007
-0.04	1	0.002	0.054	1	0.004	0	1	0	0	1	0
-0.052	0.002	1	-0.070	0.004	1	-0.052	0.006	1	-0.051	0.013	1

Table A-3. Matrices M^{-1}

Case 1			Case 2			Case 3			Case 4		
0.996	0.040	0.052	0.992	0.054	0.070	0.999	0.000	0.013	1.000	0.000	0
-0.007	-0.040	0.998	0.000	-0.054	0.997	-0.000	0	1	0	0	1
0.052	0.000	0.997	0.070	-0.000	0.995	0.052	-0.006	0.999	0.051	-0.013	1.000

Table A-4 presents elements of the matrix B, in its transposed form. From (A-15), most of the sub-matrices in B are identically zero; only the values of those submatrices that are not identically zero are presented in table A-4. Also, it should be noted that each entry in B is identical to a value of the matrix M for the corresponding case.

Table A-4. Non-Zero Elements of the Submatrices of B^T

Element of B	Element of M	Case 1	Case 2	Case 3	Case 4
B^T_{11}	M_{11}	1	1	1	1
B^T_{21}	M_{12}	-0.0403	-0.054	0	0
B^T_{31}	M_{13}	0.0524	0.070	0.0130	0.0073
B^T_{42}	M_{21}	0.0402	0.054	0	0
B^T_{52}	M_{22}	1	1	1	1
B^T_{62}	M_{23}	0.0021	0.004	0	0
B^T_{73}	M_{31}	-0.0524	-0.070	-0.0520	-0.0508
B^T_{83}	M_{32}	0.0021	0.004	0.0063	0.0130
B^T_{93}	M_{33}	1	1	1	1

The convention for integer values of zero and one, used above, is again used in table A-4.

The four sensitivity coefficient matrices, S, are presented below, as (A-19-1), through (A-19-4), for cases 1 through 4, respecti ely, for the linear survey errors. They are presented as matrix equations, rather than as tables or figures, to facilitate use and interpretation. The vector of survey errors $\underline{\varepsilon}$ is presented, in the transposed form, above the matrix S for Case 1, for clarity. This vector, in its proper form, not transposed, post-multiplies the matrix S, as shown above.

$$\underline{\varepsilon}^T = (\ \Delta x_D \quad \Delta y_D \quad \Delta z_D \quad \Delta x_A \quad \Delta y_A \quad \Delta z_A \quad \Delta x_E \quad \Delta y_E \quad \Delta z_E\ )$$

Case 1

$$S = \begin{matrix} 0.996 & -0.040 & 0.052 & 0.002 & 0.040 & 0.000 & 0.003 & 0.000 & -0.052 \\ -0.040 & 0.002 & -0.002 & 0.040 & 0.998 & 0.002 & 0.000 & 0.000 & 0.000 \\ 0.052 & -0.002 & 0.003 & 0.000 & 0.000 & 0.000 & -0.052 & 0.002 & 0.997 \end{matrix} \tag{A-19-1}$$

Case 2

$$S = \begin{matrix} 0.992 & -0.054 & 0.070 & 0.003 & 0.054 & 0.002 & 0.005 & 0.000 & -0.070 \\ -0.054 & 0.003 & -0.004 & 0.054 & 0.997 & 0.004 & 0.000 & 0.000 & 0.000 \\ 0.070 & -0.004 & 0.005 & 0.000 & 0.000 & 0.000 & -0.070 & 0.004 & 0.995 \end{matrix} \tag{A-19-2}$$

Case 3

$$S = \begin{matrix} 0.999 & 0.000 & 0.013 & 0.000 & 0.000 & 0.000 & 0.001 & 0.000 & -0.016 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.052 & 0.000 & 0.001 & 0.000 & -0.006 & 0.000 & -0.052 & 0.006 & 1.000 \end{matrix} \qquad \text{(A-19-3)}$$

Case 4

$$S = \begin{matrix} 1.000 & 0.000 & 0.007 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.051 & 0.000 & 0.000 & 0.000 & -0.013 & 0.000 & -0.051 & 0.013 & 1.000 \end{matrix} \qquad \text{(A-19-4)}$$

In the MMLS, the DME is physically non-separable from the azimuth antenna. Consequently, an error of the survey for the DME location will be exactly repeated for the azimuth antenna. This physical constraint may be expressed in the mathematics by adding columns 1, 2, and 3 of the above matrices, representing the sensitivity of aircraft location estimate to DME survey errors, to columns 4, 5, and 6, respectively, representing the sensitivity to azimuth antenna survey errors. Then columns 1, 2, and 3 may be omitted, and the multipliers of columns 4-6 must be interpreted as "x, y, and z-components of the joint survey error of the position of the DME/azimuth combination", now subscripted (DA). This procedure reduces (A-19-1) through (A-19-4) to (A-20-1) through (A-20-4), preceded by the appropriately modified vector (transposed) of the survey errors. Further, $\rho_A = \rho_D = \rho_{DA}$, to keep the notation consistent with the fact of collocation, as remarked above.

$$\underline{\varepsilon}^T = (\Delta x_{DA} \quad \Delta y_{DA} \quad \Delta z_{DA} \quad \Delta x_E \quad \Delta y_E \quad \Delta z_E)$$

Case 1

$$S = \begin{matrix} 0.998 & 0.000 & 0.052 & -0.003 & 0.000 & -0.052 \\ 0.000 & 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.052 & -0.002 & 0.003 & -0.052 & 0.002 & 0.997 \end{matrix} \qquad \text{(A-20-1)}$$

Case 2

$$S = \begin{matrix} 0.995 & 0.000 & 0.072 & 0.005 & 0.000 & -0.070 \\ 0.000 & 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.070 & -0.004 & 0.005 & -0.070 & 0.004 & 0.995 \end{matrix} \qquad \text{(A-20-2)}$$

Case 3

$$S = \begin{matrix} 0.999 & 0.000 & 0.013 & 0.001 & 0.000 & -0.016 \\ 0.000 & 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.052 & -0.006 & 0.001 & -0.052 & 0.006 & 1.000 \end{matrix} \qquad \text{(A-20-3)}$$

Case 4

$$S = \begin{matrix} 1.000 & 0.000 & 0.007 & 0.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & -0.013 & 0.000 & -0.051 & 0.013 & 1.000 \end{matrix} \qquad \text{(A-20-4)}$$

But there remains one last subtlety, pertinent to the collocated situations of Cases 1 and 2. When the three units are collocated, a survey error in Δx_A appears not only in Δx_D due to the physical connection between the azimuth antenna and the DME, it could also appear in Δx_E, if the surveyor did not measure Δx_E separately but let the fact of collocation serve to establish the identities; so also with Δy_A and Δz_A. This behavior would be expressed mathematically by adding columns 1, 2, and 3, to columns 4, 5, and 6, respectively, in (A-20 1) through (A-20-4). However, the survey might be conducted separately for the elevation antenna, since the elevation antenna is not physically attached to the other units. This latter viewpoint was taken as the rule for the several cases.

In understanding and using (A-20- 1) through (A-20-4) it is important to appreciate that almost all of the sensitivity coefficients are either zero or very small. The significance of a very small coefficient is that a large error in the survey has a very small effect on the aircraft location estimate when the aircraft is at the selected DH. As an example, the element S_{13} in Case 1 (A-20-1) is 0.052; this means that a survey error of 10' in the z-direction produces an error of 0.52' in the x-direction component of the aircraft location estimate. Contrast this with the value $S_{22} = 1.000$ in the same matrix; this coefficient implies that a survey error of 10' in the y-direction produces an error of 10' in the y-component of aircraft location estimate. The former piece of information shows that the error is very insensitive to that particular variable, which is thus not useful for setting survey accuracy requirements, and may thus be neglected with small effects. On the other hand, the latter result is important in the present context, and provides some basis for setting the tolerable survey error limits.

Neglecting all coefficients of magnitude less than or equal to 0.020 enables reducing the four sensitivity-coefficient matrices in (A-20-1) through (A-20-4) to four simpler, but essentially equivalent, equations, in (A-21-1) through (A-21-4). The situation is, in all four cases, still mathematically underdetermined. However, neglecting the parameters with extremely small coefficients enables clarifying and simplifying the procedure, and reduces the problem of treating eleven parameters, which is unncessarily complicated, to a problem with two major terms, or less, in each inequality constraint. This simpler problem can successfully be treated by using judgement to meet and to establish the budgets.

Case 1

$$\begin{aligned}
\Delta x_T &= 0.998\Delta x_{DA} + 0.052\Delta z_{DA} - 0.052\Delta z_E \\
\Delta y_T &= \Delta y_{DA} \\
\Delta z_T &= 0.052\Delta x_{DA} - 0.052\Delta x_E + 0.997\Delta z_E
\end{aligned} \tag{A-21-1}$$

Case 2

$$\begin{aligned}
\Delta x_T &= 0.995\Delta x_{DA} + 0.072\Delta z_{DA} - 0.070\Delta z_E \\
\Delta y_T &= \Delta y_{DA} \\
\Delta z_T &= 0.070\Delta x_{DA} - 0.070\Delta x_E + 0.995\Delta z_E
\end{aligned} \tag{A-21-2}$$

Case 3

$$\Delta x_T = 0.999\Delta x_{DA}$$
$$\Delta y_T = \Delta y_{DA} \qquad \text{(A-21-3)}$$
$$\Delta z_T = 0.052\Delta x_{DA} - 0.052\Delta x_E + \Delta z_E$$

Case 4

$$\Delta x_T = \Delta x_{DA}$$
$$\Delta y_T = \Delta y_{DA} \qquad \text{(A-21-4)}$$
$$\Delta z_T = 0.051\Delta x_{DA} - 0.051\Delta x_E + \Delta z_E$$

The alignment effects are now considered. Using the geometry yields the contributions of the alignment errors to the setup error:

Lateral: Azimuth antenna alignment

$$\Delta y \approx (\rho_{DA}\cos\theta)\ \{\sqrt{1 - (\sin\phi_A/\cos\theta_{PE})^2}\ \Delta\alpha_Y - (\sin\phi_A/\cos\theta_{PE})\Delta\alpha_R)\} \qquad \text{(A-22-1)}$$

where ϕ_A is the aircraft's elevation angle relative to the azimuth antenna, instead of the elevation antenna, and θ_{PE} is the aircraft's planar (not conical) azimuth angle relative to the elevation antenna.

Vertical: Elevation antenna alignment

$$\Delta z = (\rho_E\cos\phi_A)\{ -(\sin\theta_{PE})\Delta\beta_R + (\cos\theta_{PE})\Delta\beta_P\} \qquad \text{(A-22-2)}$$

Under the usual small angles assumptions, these alignment expressions simplify to

$$\Delta y \approx (\rho_A)\ \{\Delta\alpha_Y - (\sin\phi_A)\Delta\alpha_R\} \qquad \text{(A-23-1)}$$

$$\Delta z \approx (\rho_E)\{ -(\sin\theta_{PE})\Delta\beta_R + \Delta\beta_P\} \qquad \text{(A-23-2)}$$

After the alignment is completed, $\Delta\alpha_Y$ is controlled by a monitor, and the error of this mechanism is contained within the MCE; this term may therefore be omitted, since it does not contribute an error not otherwise considered. However, an azimuth alignment error exists; it is considered at the end of this appendix. Similarly, the pitch of the elevation antenna, $\Delta\beta_P$, is controlled by an internal levelling circuit, and this term may also therefore be omitted. Then (A-23) become

$$\Delta y \approx (\rho_{DA})\ \{- (\sin\phi_A)\Delta\alpha_R\} \qquad \text{(A-24-1)}$$
$$\Delta z \approx (\rho_E)\{ -(\sin\theta_{PE})\Delta\beta_R\ \} \qquad \text{(A-24-2)}$$

The terms ρ_{DA} and ρ_E are slant ranges; their values are given in table A-1. Converting (A-24) to degrees and using the values in table A-1 yields the newly defined auxiliary angles in table A-5.

Table A-5. Auxiliary Angles for Computing Alignment Error Terms

ϕ_A	3°	4°	0.747°	0.418°
θ_{PE}	2.311°	3.084°	6.947°	14.374°

These new angles, and the geometry defined in table A-1, are now used to determine the coefficients of the alignment terms in (A-24). These alignment error terms appear in table A-6.

Table A-6. Incremental Setup Errors Due to Alignment Errors

	Case 1	Case 2	Case 3	Case 4
Lateral	$3.40\Delta\alpha_R$	$3.40\Delta\alpha_R$	$3.40\Delta\alpha_R$	$1.66\Delta\alpha_R$
Vertical	$-2.62\Delta\beta_R$	$-2.62\Delta\beta_R$	$7.86\Delta\beta_R$	$7.86\Delta\beta_R$

These results may now be combined with (A-21), yielding the final result. The following expressions give the error in feet of the aircraft position due to errors in feet due to survey data, and due to errors in degrees due to the angular orientation of the antennas.

Case 1

$$\begin{aligned} \Delta x_T &= 0.998\Delta x_{DA} + 0.052\Delta z_{DA} - 0.052\Delta z_E \\ \Delta y_T &= \Delta yDA + 3.40\Delta\alpha_R \\ \Delta z_T &= 0.052\Delta x_{DA} - 0.052\Delta x_E + 0.997\Delta z_E - 2.62\Delta\beta \end{aligned} \tag{A-25-1}$$

Case 2

$$\begin{aligned} \Delta x_T &= 0.995\Delta x_{DA} + 0.072\Delta z_{DA} - 0.070\Delta z_E \\ \Delta y_T &= \Delta y_{DA} + 3.40\Delta\alpha_R \\ \Delta z_T &= 0.070\Delta x_{DA} - 0.070\Delta x_E + 0.995\Delta z_E - 2.62\Delta\beta_R \end{aligned} \tag{A-25-2}$$

Case 3

$$\Delta x_T = 0.999\Delta x_{DA}$$
$$\Delta y_T = \Delta y_{DA} + 3.40\Delta\alpha_R \qquad \text{(A25-3)}$$
$$\Delta z_T = 0.052\Delta x_{DA} - 0.052\Delta x_E + \Delta z_E + 7.86\Delta\beta_R$$

Case 4

$$\Delta x_T = \Delta x_{DA}$$
$$\Delta y_T = \Delta y_{DA} + 1.66\Delta\alpha_R \qquad \text{(A-25-4)}$$
$$\Delta z_T = 0.051\Delta x_{DA} - 0.051\Delta x_E + \Delta z_E + 7.86\Delta\beta_R$$

In determining the error to be allowed for the survey and alignment, it will be arbitrarily assumed that these survey and alignment errors all are in the precisely wrong directions; i. e., that they are correlated with correlation coefficients of -1 as appropriate to create the worst case. This is equivalent to replacing the terms in (A-25-1) through (A-25-4) by their magnitudes.

Summarizing this appendix to this point:

a. A theoretical development has been formed of the relationships between the geometrical elements of the survey, the alignments of the antennas, and the errors of aircraft location estimate in computed centerline operation;

b. These results have been converted to yield the sensitivity coefficients which specify the error of position estimate per unit of survey or alignment error;

c. The numerical elements of the geometry have been derived and organized to enable computation of the sensitivity coefficients;

d. The sensitivity coefficients have been evaluated. The fact that the DME is built in as a part of the azimuth antenna has been accounted for, and the results have been simplified to eliminate trivial terms;

e. The effects of angular alignment errors have been included.

The consequent numerical results are now available for the analyses of sections 3 through 6 of the text.

A.3 SENSITIVITY OF RESULTS

The sensitivity of the relationships that have been developed above to the assumed geometry is now considered. Inspection of the definitions of the matrices M and B shows that their terms are not sensitive to small changes of the geometry. As the values of the four determinants of the matrices M are near unity, the matrix inverses are not sensitive to the

geometry. Similarly, inspection of the angular alignment terms shows that they are not sensitive to small changes of the geometry.

It should be observed that the aircraft is assumed to be at a specified location in each part of the analyses to enable computation of the various error-sensitivity coefficients. This is not strictly correct for the aircraft lateral position assumption in Cases 1 and 2. In these two cases, in which the azimuth antenna is offset from the runway centerline, the DME error of slant range interacts with the geometry to produce a lateral error. This error interacts with the guidance so that the aircraft is not exactly on the centerline, as assumed for the analysis, if the DME range error is non-zero. The effect of the interaction of DME error and lateral offset of the azimuth antenna on the aircraft position is discussed in appendix B. But, again, inspection of the relationships, here and in appendix B, shows that the coefficients are insensitive to small changes of aircraft location in any direction. Therefore this failure of exact compliance with the assumptions is not important.

Similarly, the assumptions concerning the random sources of error do not introduce large sensitivities. This point is considered further. Assume that the random errors, combined RSS, total to 10 percent of the given tolerance of the window; then the margin for the survey errors is 90 percent of the window size, and small changes in the random errors have a minute effect on the available margin. Conversely, if the random errors constitute 90 percent of the window, and only 10 percent remains for the survey, then the sensitivity of the survey allowances to the random errors is very high. This perception may be given rigorous form.

Define

R	RSS of all random errors
W	Window size
A	Available margin for survey errors

Then, as $A = W - R$, the fractional sensitivity of A, $(\Delta A/A)$, is related to the fractional sensitivity of R, $(\Delta R/R)$, as

$$(\Delta A/A) = -\,[\,(R/W)/(1 - R/W)]\,(\Delta R/R) = -\,F\,(\Delta R/R) \qquad \text{(A-26)}$$

where the sensitivity ratio, F, is

$$F = [\,(R/W)/(1 - R/W)] \qquad \text{(A-27)}$$

Table A-6 shows the relationship, from (A-27), between F, the fractional sensitivity of A, and R/W. If, for example, the random errors are 30 percent of the window size, so that (R/W) = 0.3, then a 10 percent change of (R/W) produces a 4.3 percent change in the margin available for the setup errors. But if (R/W) = 0.6, then a 10 percent change in (R/W) produces a 15 percent change in the margin available for the survey and alignment errors.

Table A-6. Sensitivity of Survey Margin to Random Errors

(R/W)	0	0.1	0.2	0.3	0.4	0.5	0.6
F	0	0.11	0.25	0.43	0.67	1.0	1.5

The only assumption that can have an important effec has to do with human error. It is necessary to assume that the setup team has indeed conducted the setup procedure correctly, that when the antenna is believed to be aimed at Monitor A it is not aimed at Monitor B, that the Monitor on the runway centerline is at least near enough to the centerline so the MCE criterion is not violated, that gross errors of data entry have not been made, etc. Without this assumption of approximately correct performance by the setup crew, it is impossible to conduct an analysis, and is, equally, impossible to use or deploy the system.

A-4. AZIMUTH ANTENNA YAW ALIGNMENT

Below (A-23) it was noted that an azimuth error exists due to yaw alignment; this topic is now considered. The critical cases for yaw alignment are Cases 1 and 2, wherein the azimuth antenna is offset from the runway centerline, and the centerline is not available as a reference. The setup procedure first positions the azimuth antenna or its marker, offset from the runway centerline if desired. The procedure then positions the azimuth survey sighting pole, or its marker, presumably at the same offset distance, and approximately 500' downwind from the antenna. The azimuth antenna is then positioned and mechanically rotated in yaw until its optical sight, aligned with its electronic boresight, is centered on the pole. If the line of sight from the antenna phase center to the pole is not parallel to the runway centerline, then the azimuth antenna has a yaw error. This yaw error produces an error of calculated position of the aircraft at the various locations. This error of calculated position is now determined as a function of the errors of emplacement of the azimuth antenna and its sighting pole.

Define

Δy_A	Lateral Position Error of azimuth antenna
Δy_P	Lateral Position Error of azimuth antenna sighting pole
Δy_{PA}	Total Error to be allowed for the combined effects of lateral survey error of the antenna and of the sighting pole.

If the sighting pole is 500' from the antenna, it is not realistic to use the antenna as a reference for positioning the pole; it has to be positioned by a separate act of surveying; this is especially the case when the azimuth antenna is offset from the centerline. Therefore, the antenna and the pole locations must be determined by separate surveys, each of which has the same error possibilities, so that these two error sources (Δy_P and Δy_A) must have the same error statistics. Further, the sum of the effects of these two errors may not exceed the allowable part, Δy_{PA}, of the margin.

Now consider figure A-2. This figure shows the total lateral error of the estimated position of the aircraft, Δy_{PA}, due to the errors of the positioning of the azimuth antenna and the sighting pole. From this diagram, by using a small angles assumption, we deduce that

$$\Delta y_{PA} \geq \Delta y_P(\rho_{DA} / 500) - \Delta y_A \qquad \text{(A-28)}$$

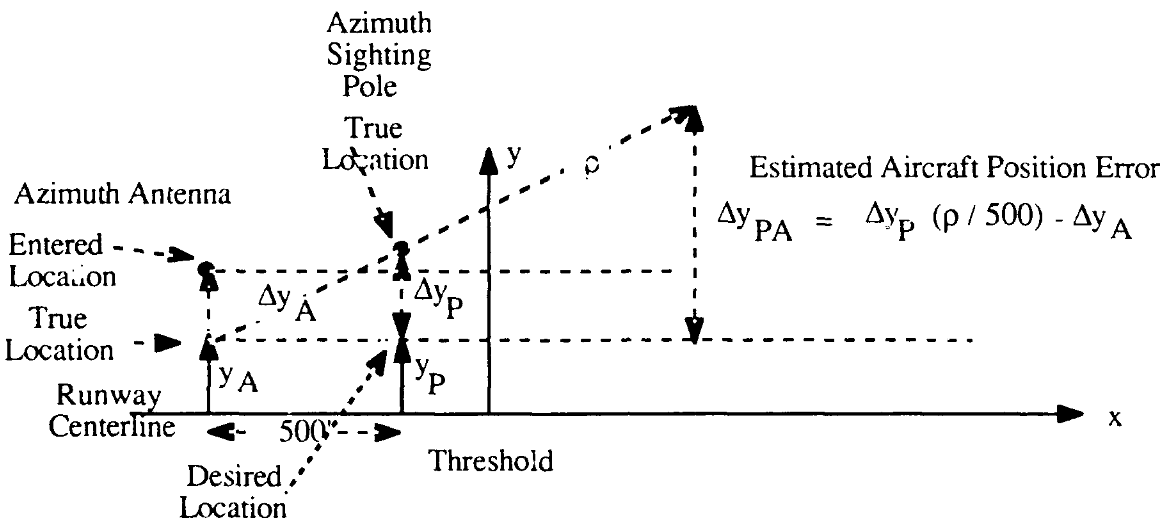

Figure A-2. Effects of Positioning Errors of the Azimuth Antenna and Its Siting Pole

This expression embodies the second and final remark in the preceding paragraph. The first two sentences of that paragraph imply that the two error sources are of equal magnitudes (or standard deviations, etc.), so that $|\Delta y_A| = |\Delta y_P|$. Combining these relationships, (A-28) yields $\Delta y_{PA} \geq \{|\Delta y_P| (\rho_{DA} / 500) + |\Delta y_P|\}$. This may be solved to yield the allowable error budget for the siting accuracy of the pole and also of the azimuth antenna as

$$|\Delta y_P| = |\Delta y_A| = \leq \Delta y_{PA} / (1 + \rho_{DA} / 500) \qquad \text{(A-29)}$$

This result will be used in sections 3 through 6. Notice that $(-\Delta y_A)$ in (A-28) changes to $(+ |\Delta y_P|)$ in the steps leading to (A-29) due to the use of magnitudes, and because the positioning errors of the antenna and the pole are independent.

APPENDIX B

INTERACTION OF THE DME ERROR AND GUIDANCE IN CASES 1 AND 2

Near the end of appendix A, it was noted that the error of the DME interacts with the offset location of the azimuth antenna and with the guidance system to produce a lateral position error of the aircraft. This effect is now examined.

Notation special to this section is defined below.

Notation

$\Delta\rho$	DME range error
y_I	Total lateral indicated position relative to centerline due to DME range error
ε_D	Error of lateral position due to DME error, including guidance effects

Assumptions

1. Altitude effects may be neglected so that $z_T = z_E$, and there are no piloting errors;
2. The airplane follows the path of zero indicated error, so that $y_I = 0$;
3. The ground units are collocated at the El site.

Analysis to Find Flight Path

Under assumption 1, we have the relationship, shown in figure B-1,

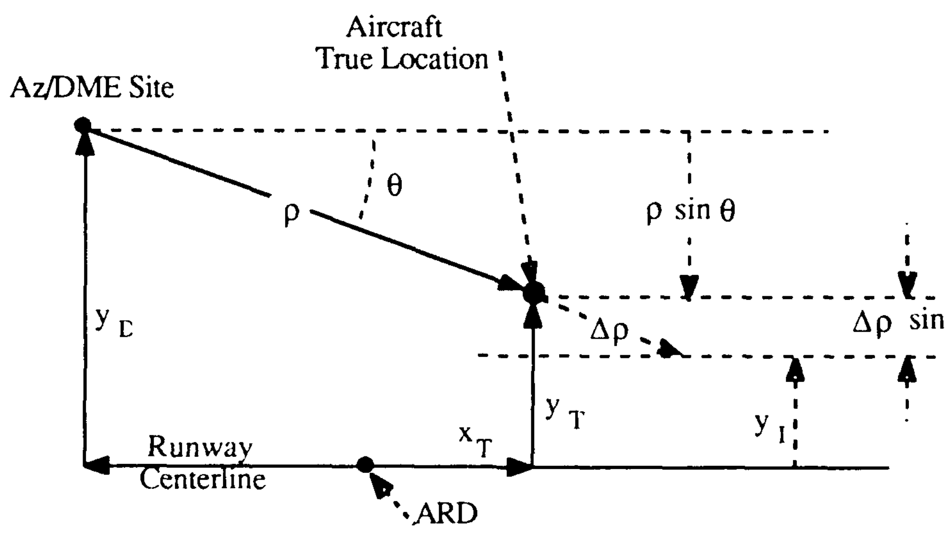

Figure B-1. Geometry for Analysis of Effect of DME Error

$$y_I = y_D - \rho\sin\theta - \Delta\rho\sin\theta \quad \text{(B-1)}$$

The azimuth angle is defined by

$$\sin\theta = -(y_T - y_D)/\rho \quad \text{(B-2)}$$

Substitute (B-2) into (B-1) to find

$$y_I = y_D + [(\rho + \Delta\rho)(y_T - y_D)/\rho] \quad \text{(B-3)}$$

and use assumption 2 to find

$$0 = y_D + (\rho + \Delta\rho)(y_T - y_D)/\rho \quad \text{(B-4)}$$

In (B-4), gather the factors of y_T and y_D, and then solve for y_T. This yields

$$y_T = y_D[\,1 - \rho/(\rho + \Delta\rho)] \quad \text{(B-5)}$$

As the pilot is, presumably, flying the aircraft so that the indicated lateral position, y_I, is zero, then the aircraft true lateral position is the error and consequently

$$y_T = \varepsilon_D = y_D[\Delta\rho/(\rho + \Delta\rho)] \quad \text{(B-6)}$$

For this analysis, $\Delta\rho$ may be taken as zero-mean, normal, and uncorrelated with the other error sources.

Numerical Analysis

Evaluating (B-5) with the parameters of Case 1 from table A-1, and $\Delta\rho = 644'$ as shown below (3-2) in section 3, yields the lateral position error due to the DME range-error as

$$\varepsilon_D = |[644(150)/(3725 + 644)]| = 22.1' \quad \text{(B-7)}$$

and

$$\varepsilon_D = |[-644(150)/(3725 - 644)]| = 31.4' \quad \text{(B-8)}$$

Assume that the range error and the azimuth antenna lateral offset are both positive. Then it is important to appreciate that the lateral error of the aircraft position due to this effect is also positive.

GLOSSARY

ARD	Approach Reference Datum
Cat	Category of Landing
DH	Decision Height
ICAO	International Civil Aeronautics Association
MCE	Mean Course Error
MGE	Mean Glide Slope Error
MLS	Microwave Landing System
MMLS	Mobile Microwave Landing System
PFE	Path Following Error
PFN	Path Following Noise
RNAV	Area Navigation
RSS	Root Sum Squared
TACAN	Tactical Air Navigation